SUNBLAZER RECEIVER PERFORMANCE FOR A NOISY, DISPERSIVE CHANNEL

by

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ABSTRACT

The performance of the Sunblazer phase modulation communication scheme is examined with regard to signal detection. Specifically, the optimum performance threshold is determined, and the statistics for the error probabilities (P_M and P_F) are derived. The total probability of error is then calculated for the case of an eleven-bit Barker Code as a function of four system parameters: the a priori probability that the signal is present, the signal-to-noise ratio, the dispersion ratio, and the correlation miss time. These results show that when the operating conditions are poor it is often better to base the decision as to the signal's presence only on the a priori knowledge, without even observing the correlation output. It is shown how the threshold could be better selected if the dispersion ratio and miss time could be predicted in advance.

I. INTRODUCTION

The purpose of this report is to examine in some detail the performance of a possible communication scheme for the Sunblazer radio propagation experiment. Naturally, the success of any communication scheme is dependent on the characteristics of the channel through which the signal will propagate. In the case of the solar corona, modeling the channel is difficult and depends upon assumptions based on data of dubious accuracy. The phase-modulation scheme chosen some time ago for Sunblazer and described in this report was dependent on the assumption that the time rate of density fluctuations in the coronal plasma (resulting in phase fluctuations in the signal) would be slow compared at least to the duration of the signal. More recent data has shown that this probably will not be the case, so that the advisability of using phase-modulation is questionable. Nonetheless, it is still very worthwhile to examine this scheme, since in many ways its behavior will be similar to that of alternative schemes.

We should at this point emphasize that in this report we will be interested only in one particular aspect of the communication problem—namely, detection of the received signal. This is of primary importance. The success of the Sunblazer venture depends first upon our ability to detect the signal with high probability.

Many of the ideas in this report are based on results derived previously by the authors, and that appear in earlier reports.[1,2] If the reader is interested in following the theory developed here, we suggest that he first read, or at least skim, this earlier work.

A. Review of the Sunblazer Communication Problem.

A primary purpose of the Sunblazer solar probe is to make measurements of the electron density of the extended solar corona (Refs. 1, 2, 4, 5). This is done by transmitting two narrowband signals from the satellite at two different frequencies. Each signal will experience a different propagation delay through the corona due to the frequency-dependent group velocities, which in turn are functions of the electron density. The aim, then, is to measure as accurately as possible the arrival time of a received signal.

We can view this in terms of the communication system modeled in Figure 1. The transmitted signal is:

$$x(t) = \sqrt{2E_t}\, s(t)\cos\omega_0 t = \mathrm{Re}[\sqrt{2E_t}\, s(t)e^{j\omega_0 t}] \tag{1}$$

where s(t) is narrow-band and normalized to unit energy, i.e.

$$\int_{-\infty}^{\infty} s^2(t)dt = 1$$

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[1] Mannos, J.L., "A Study of Decision Region Receivers for the Sunblazer Space Experiment", CSR Technical Report, June, 1967.

[2] Pindyck, R. S., "Reception of Dispersed Barker Codes", CSR Technical Report #T-67-1, May, 1967.

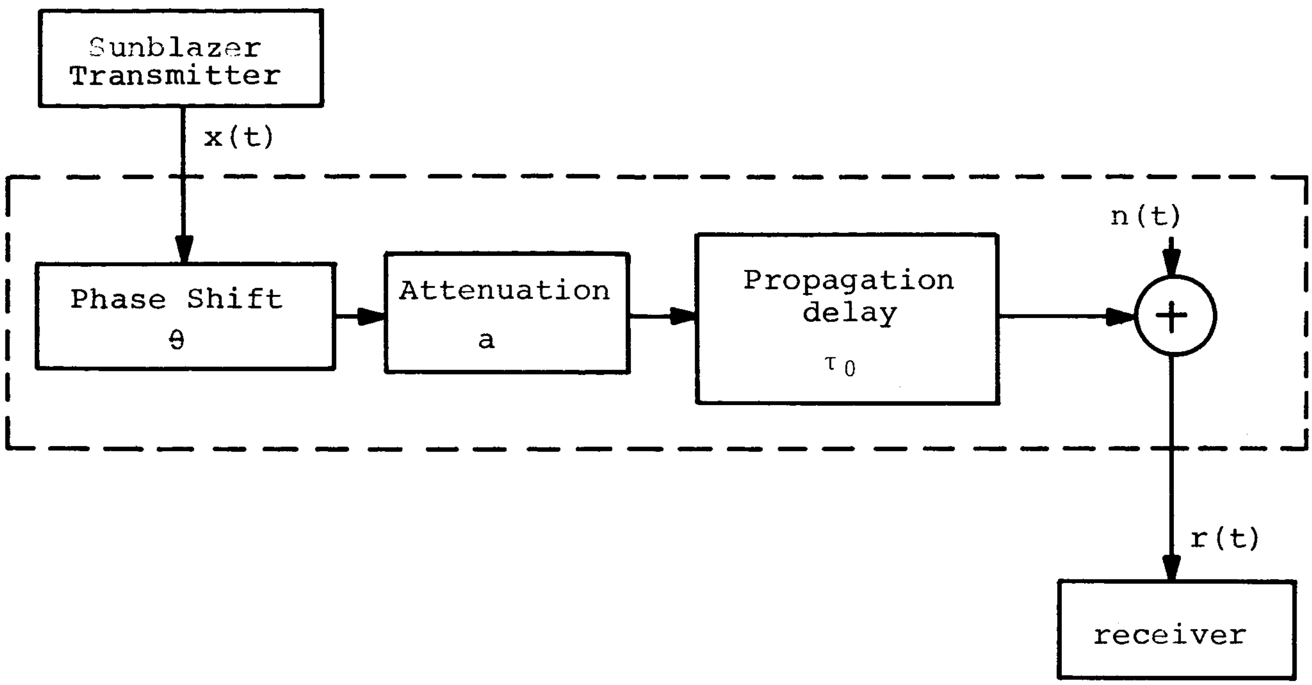


FIGURE 1 - MODEL OF THE COMMUNICATION SYSTEM

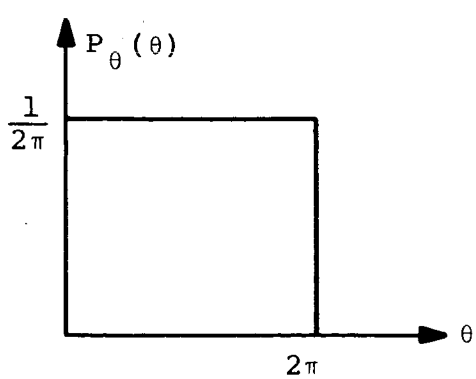


FIGURE 2 - PROBABILITY DENSITY FOR RANDOM PHASE

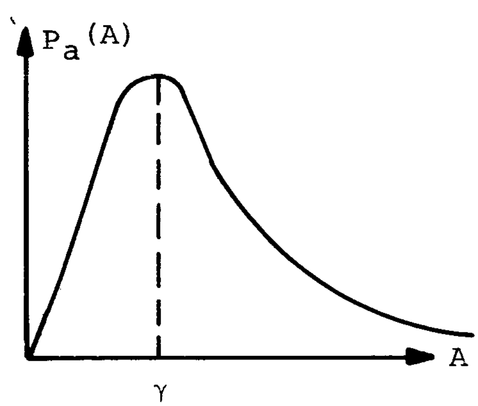


FIGURE 3 - PROBABILITY DENSITY OF RANDOM AMPLITUDE

The random phase, θ, is modeled by a uniform probability density, as in Figure 2. The random amplitude, a, is modeled by a Rayleigh density as in Figure 3. The propagation delay (or arrival time), τ_0, is treated as a real (non-random) but unknown parameter. Finally, the additive noise, n(t), is modeled as white Gaussian with variance $N_0/2$. The received signal is then:

$$r(t) = a\sqrt{2E_t}\, s(t - \tau_0)\cos(\omega_0 t + \theta) + n(t) \qquad (2)$$

Note that dispersion has been ignored in our model for the channel. We are interested in the receiver that is designed to perform optimally for a dispersionless channel. Later we will see how the performance of this receiver is affected when dispersion is added to the channel.

The communication problem consists of two parts. First we must detect the signal, i.e. make a decision as to whether or not the signal is present, and then estimate the signal's arrival time as accurately as possible. To achieve both detection and arrival time estimation it is sufficient, and optimum, for receiver to calculate the square of the correlation function of the demodulated signal (Ref. 7, ch.7). This "correlation receiver" is shown in Figure 4. Its output is:

$$X^2(\tau) = \frac{a^2E_t}{2} R^2(\tau - \tau_0) + \text{noise terms} \qquad (3)$$

Here $R^2(\tau - \tau_0)$ is the shifted auto-correlation function of s(t). A decision as to the presence of the signal is then made based on the height of the central peak of the output, and an estimate of τ_0 is made by looking at the time-position of this central peak.

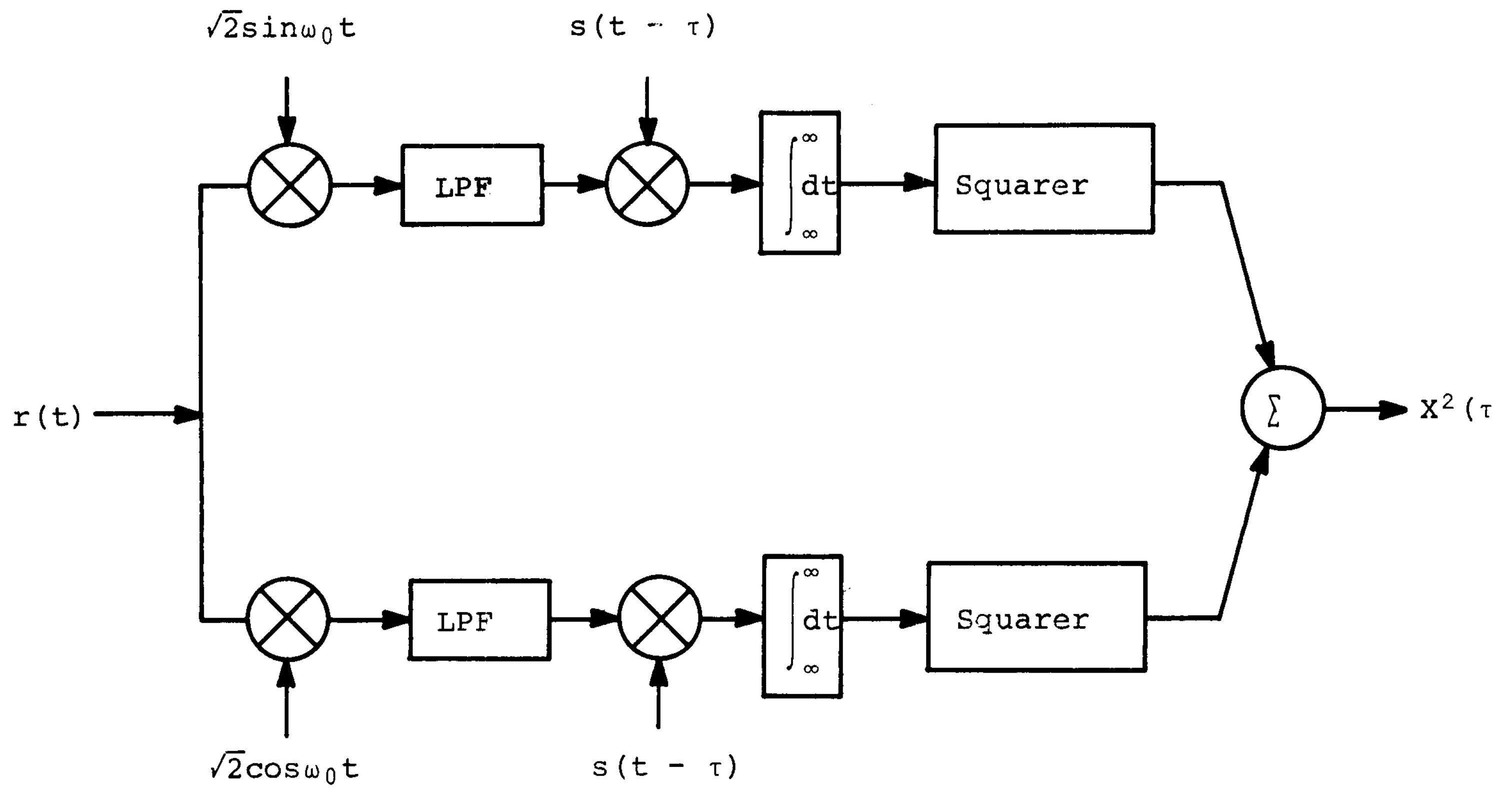


FIGURE 4 - THE CORRELATION RECEIVER

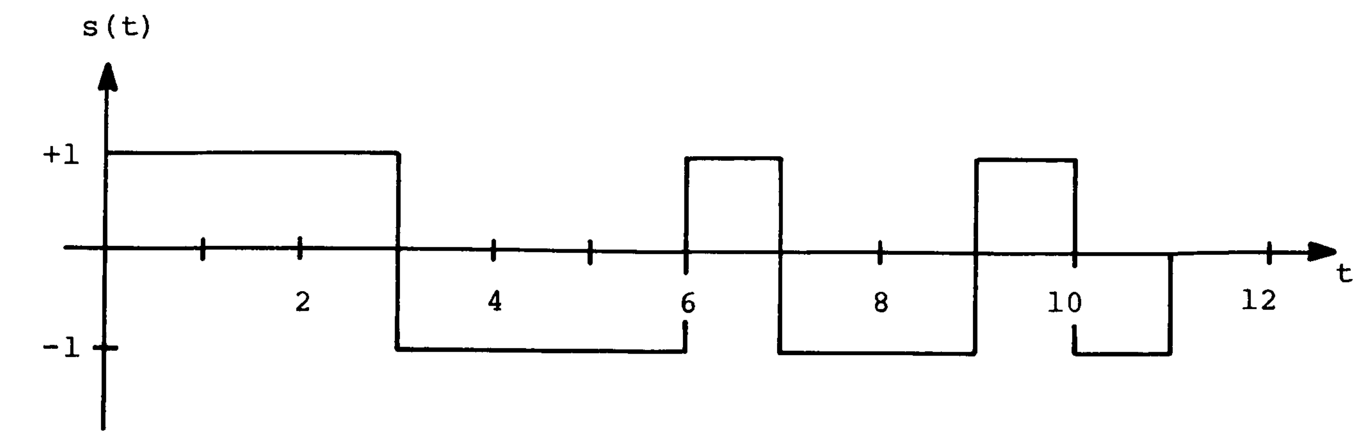


FIGURE 5a - ELEVEN-BIT BARKER CODE

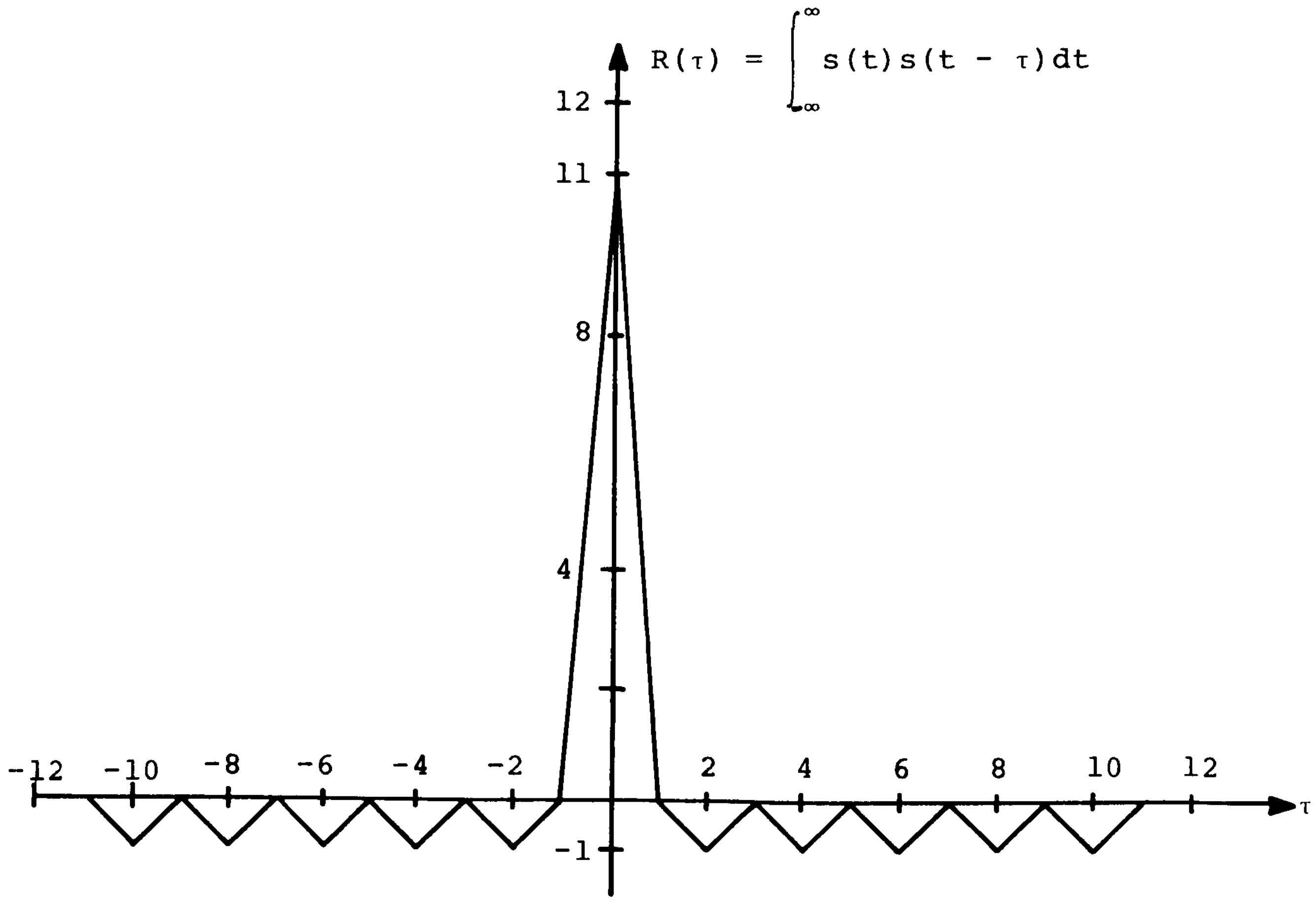


FIGURE 5b - CORRESPONDING AUTO-CORRELATION FUNCTION

Detection performance depends only on the peak value of $R(\tau)$, i.e. on signal energy, and hence cannot be improved by signal coding. Coding can, however, be beneficial with respect to estimating the arrival time. For large signal-to-noise ratio the mean-square estimation error is given by:

$$\overline{\varepsilon^2(\tau)} = \frac{1}{8\pi^2} \cdot \frac{1}{(E_r/N_0)W^2} \tag{4}$$

where W is the "effective" (mean-square) bandwidth of the signal. We would like, then, to have the bandwidth as large as possible; since the peak power is limited, we do this by coding a long, low-power signal. The eleven-bit Barker Code shown in Figure 5 gives us the desired bandwidth (and peak auto-correlation function). If necessary the signal energy can be increased (keeping the bandwidth constant) by folding the basic code on itself one or more times.

B. Signal Detection.

As mentioned previously, signal detection is achieved by looking at the peak output of the correlation receiver. If this peak is above a certain threshold, ζ, we say that a signal is present, as for example in Figure 6; otherwise we say that it is not present.

There are two types of detection errors that can occur, which we can call "miss" and "false alarm". In the case of a miss our peak correlation output is below the chosen threshold ζ, even though the signal is present. In the case of a false alarm, a peak is above the threshold though no signal is present.

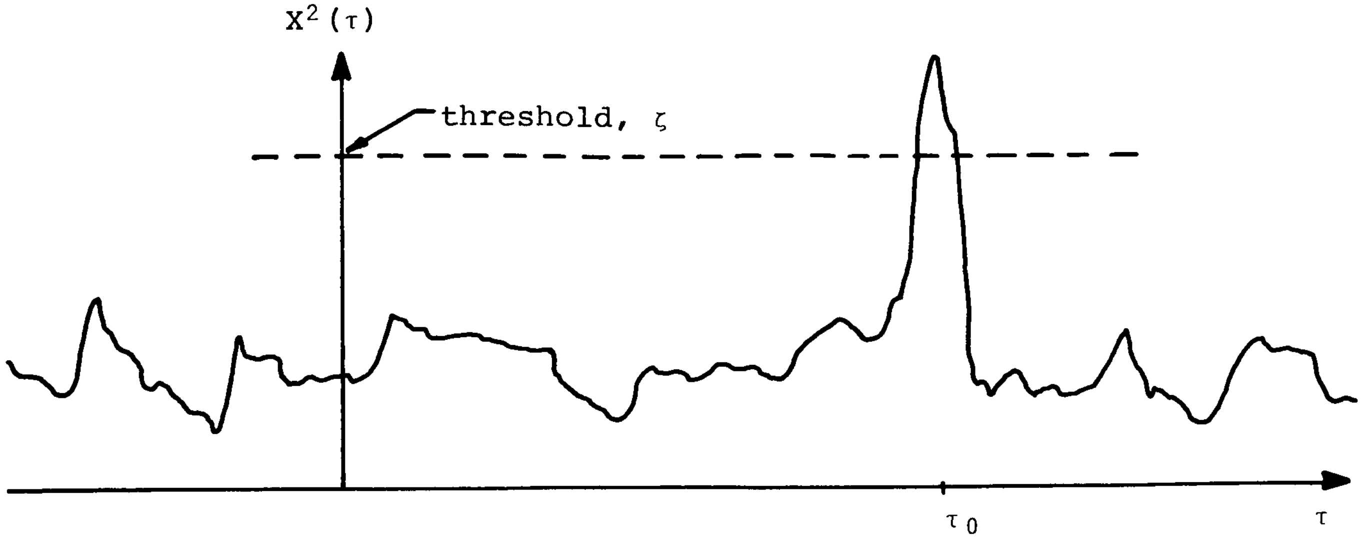


FIGURE 6 - TYPICAL OUTPUT OF CORRELATION RECEIVER

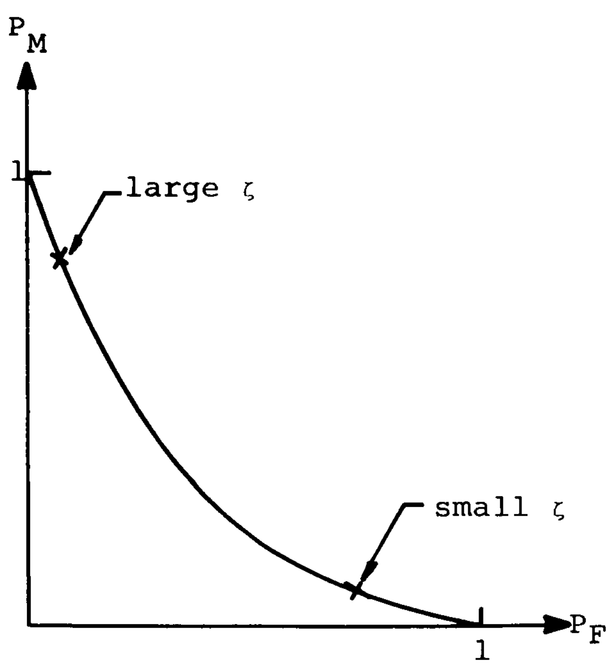


FIGURE 7a - CURVE RELATING P_M AND P_F

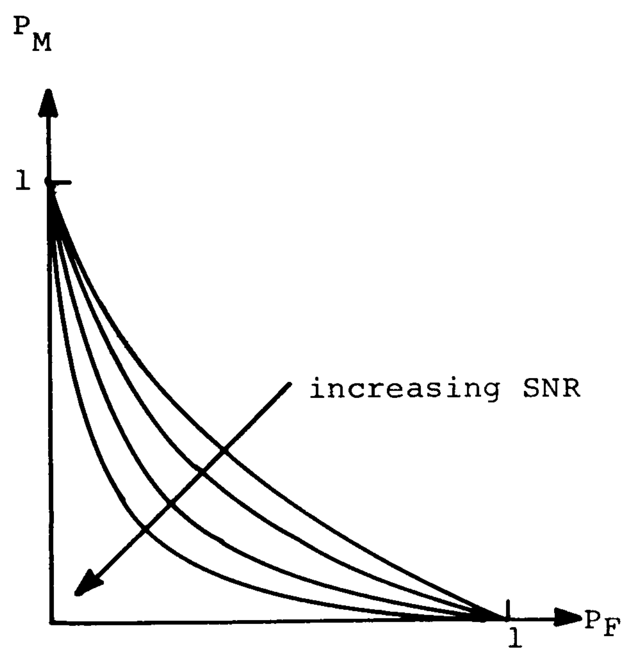


FIGURE 7b - COMPLETE ROC

The error probabilities are then:

$$P_M = Pr[\mathcal{E}|\text{signal present}] \tag{5a}$$

$$P_F = Pr[\mathcal{E}|\text{signal absent}] \tag{5b}$$

$$P_M = 1 - P_{\text{detection}} \tag{5c}$$

For a given SNR we will have a trade-off between P_M and P_F, and we could draw a curve relating the two (Figure 7a). Where we are on this curve depends on our choice of threshold ζ. In general, we could calculate an entire family of curves for different SNR that would completely describe the operation of the receiver as a detector. This set of curves (Figure 7b) is called the "receiver operating characteristic" (ROC).

The point here is that our choice of ζ, i.e. our decision rule, is somewhat arbitrary in that it depends on what kind of performance we desire. For example, we might assign a certain cost to making each type of error, and try to minimize the average cost:

$$\overline{C} = C(M)P_M + C(F)P_F$$

Here we would pick ζ so as to minimize $\overline{C}$. This average cost would then be our performance measure. We could also use the total error probability as our performance measure, namely:

$$P(\mathcal{E}) = Pr[\text{signal present}] \cdot P_M + Pr[\text{signal absent}] \cdot P_F \tag{6}$$

We will be interested in the statistics of the peak value of $X^2(\tau)$. Suppose that we could write the conditional probability densities for this peak output given that the signal is absent, and given that the signal is present (Figure 8). P_M and P_F could then be found by integrating the densities over the regions indicated. For a given SNR we would then have P_M and P_F both as a function of ζ, and could then obtain P_M in terms of P_F as desired. We leave these calculations for chapters II and III.

C. Objectives.

Our main goal is to determine how the total probability of error, $P(\mathcal{E})$, changes as we vary some of the different parameters that characterize the communication system. In the next two chapters we will examine the optimum receiver (optimum for random-phase, random amplitude, dispersion-free channel) and derive the statistics for the correlation output. This will then enable us to calculate an expression for the total probability of error. Next we will discuss dispersion and how it fits into the communication problem. At that point we will be able to calculate the ROC's, and see how $P(\mathcal{E})$ changes as a function of four parameters-signal-to-noise ratio, the "dispersion ratio" (just a convenient measure of dispersion), the a priori probability that the signal is present, and the time τ that the correlation receiver "misses" the main peak (there are only a finite number of correlation modules). This should give us a good idea of how we can expect this communication system to perform.

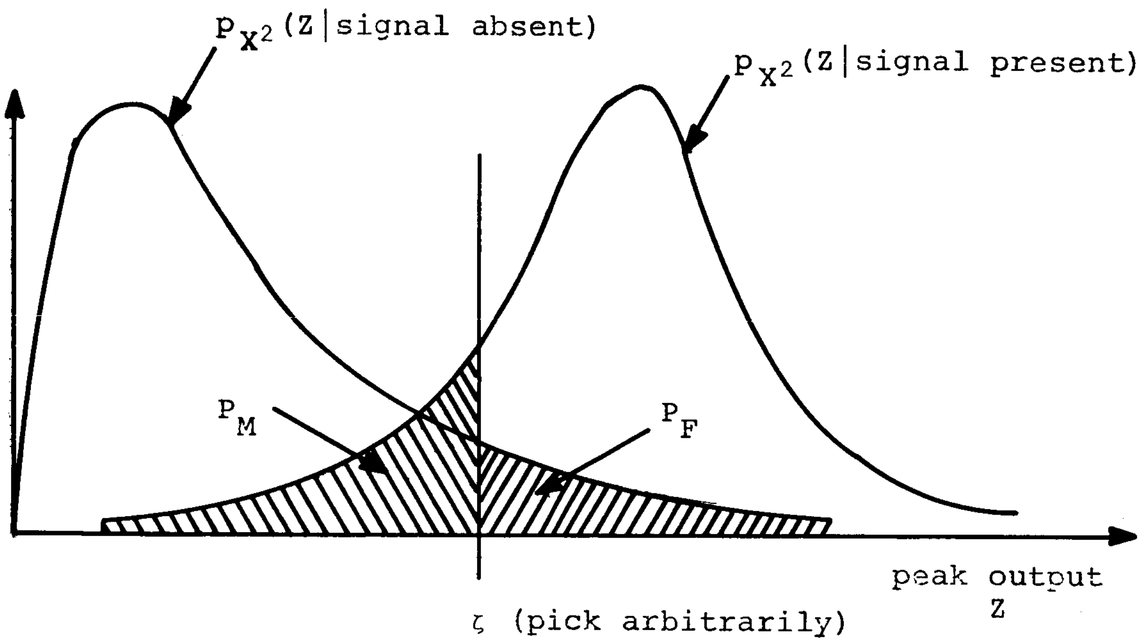


FIGURE 8 - CONDITIONAL DENSITIES FOR PEAK OUTPUT

II. THE OPTIMUM RECEIVER

As we have seen before, our receiver makes a decision as to whether or not a signal is present by comparing the peak of the correlation output to a pre-determined threshold. In this chapter we will discuss how to select the threshold so as to make our receiver "optimum". We will begin with a general definition of the optimum receiver; namely, that receiver which minimizes the Bayes Risk. To find the threshold which does this we need only evaluate the expression for the likelihood ratio test. To do this, however, we must first derive the statistics for X^2, the correlation output. Finally, we will find the threshold that minimizes the total probability of error (a special case) by setting the error costs both equal to one.

A. The Statistics for X^2.

Referring to Figure 4, we see that after demodulation (i.e. after low-pass filtering) we have in the sine and cosine channels of the correlation receiver:

$$r_c(t) = a\sqrt{E_t}\, s(t - \tau_0)\cos\theta + n_c(t) \tag{7a}$$

$$r_s(t) = a\sqrt{E_t}\, s(t - \tau_0)\sin\theta + n_s(t) \tag{7b}$$

Rewriting equation 7 and expanding the signal in an orthonormal expansion:

$$s(t) = \sum_{k=0}^{1} s_k \phi_k(t) \ ; \quad \underline{s} = (s_0, s_1)^3$$

---

[3] $s_0 = 0$ since we have noise as one of the two possible "messages"

we have the vector equations

$$\underline{n}_c = \underline{r}_c - a\sqrt{E_t}\cos\theta\underline{s}R(\tau) \tag{8a}$$

$$\underline{n}_s = \underline{r}_s - a\sqrt{E_t}\sin\theta\underline{s}R(\tau) \tag{8b}$$

Here $R(\tau)$ is the normalized autocorrelation function of $s(t)$, i.e. $R(0) = 1$; and τ is the amount of time by which the receiver misses the main peak (due, of course, to the finite number of correlators).

<u>Case 1 - signal present</u>:

When the signal is present, we have

$$p_{\underline{r}_c,\underline{r}_s}(\underline{\alpha},\underline{\beta}|\text{signal},\ \theta,\ a) =$$

$$p_{\underline{n}_c,\underline{n}_s}[\underline{\alpha} - a\sqrt{E_t}\cos\theta\underline{s}R(\tau),\underline{\beta} - a\sqrt{E_t}\sin\theta\underline{s}R(\tau)] \tag{9}$$

But $\underline{n}_c$ and $\underline{n}_s$ are Gaussian, so

$$p_{\underline{r}_c,\underline{r}_s}(\underline{\alpha},\underline{\beta}|\text{signal},\ \theta,\ a) =$$

$$\frac{1}{N_0}e^{-\frac{1}{N_0}\{[\underline{\alpha} - a\sqrt{E_t}\cos\theta\underline{s}R(\tau)]^2 - [\underline{\beta} - a\sqrt{E_t}\sin\theta\underline{s}R(\tau)]^2\}} \tag{10}$$

Now note that when $\underline{r}_c = \underline{\alpha}$, and $\underline{r}_s = \underline{\beta}$, our sufficient statistic (correlation output) is given by:

$$X^2 = (\underline{s}\cdot\underline{\alpha})^2 + (\underline{s}\cdot\underline{\beta})^2 \tag{11}$$

Substituting (8) into (11) for the case when the signal is present:

$$X^2 = (a\sqrt{E_t}\cos\theta R(\tau) + n_c)^2 + (a\sqrt{E_t}\sin\theta R(\tau) + n_s)^2 \tag{12}$$

Since n_c is Gaussian, $(\underline{s}\cdot\underline{\alpha})$ is also Gaussian with variance $N_0/2$ and mean $a\sqrt{E_t}\cos\theta R(\tau)$. Similarly, $(\underline{s}\cdot\underline{\beta})$ is also Gaussian with variance $N_0/2$ and mean $a\sqrt{E_t}\sin\theta R(\tau)$. Thus X^2 is the sum of the squares of two Gaussian random variables.

If z_1 is $N(m_1,\sigma^2)$ and z_2 is $N(m_2,\sigma^2)$, and $z = z_1^2 + z_2^2$, then the probability density for z is (Ref. 4, pps. 32-33):

$$p_z(Z) = \frac{1}{2\sigma^2}e^{-(Z + m^2)/2\sigma^2} I_0\left(\frac{Z^{1/2}m}{\sigma^2}\right); \quad Z>0 \tag{13}$$

where $m = \sqrt{m_1^2 + m_2^2}$, and $I_0(\ )$ is the zero-order modified Bessel function.

In our case $m = a\sqrt{E_t}R(\tau)$ and $\sigma^2 = N_0/2$. Substituting this into (13) gives:

$$p_{X^2}(Z|\text{signal}) =$$

$$\frac{1}{N_0}e^{-\frac{1}{N_0}[Z + a\,E_tR^2(\tau)]} I_0\left(\frac{2Z^{1/2}a\sqrt{E_t}R(\tau)}{N_0}\right); \quad \text{for } Z>0 \tag{14}$$

Now this must be averaged over the Rayleigh density for a:

$$p_a(A) = \frac{A}{\gamma^2}e^{-A^2/2\gamma^2}; \quad A\geq 0 \tag{15}$$

We have,

$$\overline{p_{X^2}(Z|\text{signal})\ a} = \int_{A=0}^{\infty} \frac{A}{\gamma^2}e^{-A^2/2\gamma^2} p_{X^2}(Z|\text{signal})\,dA \tag{16}$$

$$= \int_0^{\infty} \frac{A}{N_0\gamma^2}e^{-A^2/2\gamma^2}\left[e^{-Z/N_0}e^{-A^2E_tR^2(\tau)/N_0}\right] I_0\left[\frac{Z^{1/2}A\sqrt{E_t}R(\tau)}{N_0}\right]dA \tag{17}$$

$I_0(x)$ can be expressed in terms of the series

$$I_0(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{(k!)^2} \tag{18}$$

then (17) becomes

$$\int_0^{\infty} \frac{A}{N_0\gamma^2} e^{-A^2/2\gamma^2} \left[e^{-Z/N_0} e^{-A^2E_tR^2(\tau)/N_0} \right] \sum_{k=0}^{\infty} \frac{[Z^{1/2}A\sqrt{E_t}R(\tau)/N_0]^{2k}}{(k!)^2} dA \tag{19}$$

$$= \int_0^{\infty} \frac{A}{N_0\gamma^2} e^{-Z/N_0} e^{-A^2G} \sum_{k=0}^{\infty} \frac{[Z^{1/2}A\sqrt{E_t}R(\tau)/N_0]}{(k!)^2} dA \tag{20}$$

where $G = \dfrac{E_tR^2(\tau)}{N_0} + \dfrac{1}{2\gamma^2}$ (21)

Interchanging the integral and summation in (20):

$$\sum_{k=0}^{\infty} \frac{e^{-Z/N_0}}{N_0\gamma^2} \left[\frac{Z^{1/2}\sqrt{E_t}R(\tau)}{N_0} \right]^{2k} \cdot \frac{1}{(k!)^2} \int_0^{\infty} A^{2k+1} e^{-GA^2} dA \tag{22}$$

This integral can be solved by a straight-forward substitution, and the above expression becomes:

$$\sum_{k=0}^{\infty} \frac{e^{-Z/N_0}}{N_0\gamma^2} \left[\frac{ZE_tR^2(\tau)}{N_0^2} \right]^{k} \cdot \frac{1}{(k!)^2} \cdot \frac{k!}{2G^{k+1}} \tag{23}$$

$$= \frac{1}{2GN_0\gamma^2} e^{-\frac{Z}{N_0}\left[1 - \frac{E_tR^2(\tau)}{GN_0}\right]} \tag{24}$$

Substituting (21) into (24), multiplying out terms, and then recombining, we get as the probability density for X_2

given a signal is present:

$$\overline{p_{X^2}(Z|\text{signal})\ a} = \frac{1}{N_0 C} e^{-Z/N_0 C} \quad ; \quad Z \geq 0 \tag{25}$$

$$\text{where } C = 2(E_t/N_0)\gamma^2 R^2(\tau) + 1 \tag{26}$$

Case 2 - signal absent:

When there is no signal present, we have:

$$\underline{\alpha} = \underline{n}_c,\ \underline{\beta} = \underline{n}_s, \text{ and } X^2 = (\underline{s}\cdot\underline{\alpha})^2 + (\underline{s}\cdot\underline{\beta})^2 = n_c^2 + n_s^2 \tag{27}$$

Following a similar derivation (this time the Gaussian random variables are zero-mean) we find that:

$$p_{X^2}(Z|\text{no signal}) = \frac{1}{N_0} e^{-Z/N_0}; \quad Z \geq 0 \tag{28}$$

B. The Decision Rule.

We define the optimum receiver as that which minimizes the risk, R, given by

$$R = \sum_{i=0}^{1} \sum_{j=0}^{1} P_j C_{ij} \Pr[\text{say } i|j \text{ sent}] \tag{29}$$

In our case, of course, message 1 is the Barker code, and message 0 is no signal, i.e. only noise. P_j is the a priori probability that message j was sent, and C_{ij} is the "cost" of saying that i was sent when actually j was sent. It is easy to show (Ref. 6, ch. 1) that for our binary case the decision rule that minimizes this risk is the likelihood-ratio test given by:

$$\Lambda(\underline{r}) = \frac{p_{X^2}(Z|\text{signal})}{p_{X^2}(Z|\text{no signal})} \underset{\substack{\text{choose} \\ \text{noise}}}{\overset{\substack{\text{choose} \\ \text{signal}}}{\gtrless}} \frac{P_0(C_{10} - C_{00})}{P_1(C_{01} - C_{11})} = \zeta_0 \tag{30}$$

$$= \frac{\frac{1}{N_0 C} e^{-Z/N_0 C}}{\frac{1}{N_0} e^{-Z/N_0}} \underset{\text{noise}}{\overset{\text{signal}}{\gtrless}} \frac{P_0(C_{10} - C_{00})}{P_1(C_{01} - C_{11})} = \zeta_0$$

$$= \frac{1}{C} e^{(C-1)Z/N_0 C} \underset{\text{noise}}{\overset{\text{signal}}{\gtrless}} \frac{P_0(C_{10} - C_{00})}{P_1(C_{01} - C_{11})} = \zeta_0 \tag{31}$$

Or,

$$X^2 = Z \underset{\text{noise}}{\overset{\text{signal}}{\gtrless}} \frac{N_0 C}{C - 1} \ln \left[\frac{CP_0(C_{10} - C_{00})}{P_1(C_{01} - C_{11})} \right] = \zeta \tag{32}$$

This, then, defines our optimum receiver. We simply compare the output statistic X^2 to the threshold ζ given by equation 32.

C. <u>The Threshold that Minimizes the Total Probability of Error.</u>

If we recognize that $(E_t/N_0)\gamma^2$ is the average received SNR, i.e.

$$\frac{E_t}{N_0}\gamma^2 = \frac{E_r}{N_0} \tag{33}$$

then we can rewrite equation 26 as

$$C = 2\frac{E_r}{N_0} R^2(\tau) + 1 \tag{34}$$

Now we define

$$A = 1/(2\frac{E_r}{N_0} + 1) \tag{35}$$

and

$$B = E_t/(N_0 E_t + N_0^2/2\gamma^2) \tag{36}$$

Then if $\tau = 0$ (we set the threshold under the assumption that we will not miss the main peak, so that $R = 1$), we have

$$C = \frac{1}{A}, \text{ and } \frac{N_0 C}{C-1} = \frac{1}{B}$$

and

$$X^2 \underset{\text{noise}}{\overset{\text{signal}}{\gtrless}} \frac{1}{B}\ln\left[\frac{P_0}{P_1 A}\left(\frac{C_{10} - C_{00}}{C_{01} - C_{11}}\right)\right] = \zeta \tag{37}$$

To minimize the total probability of error we simply set

$$C_{00} = C_{11} = 0$$

and

$$C_{01} = C_{10} = 1$$

The decision rule is then

$$X^2 \underset{\text{noise}}{\overset{\text{signal}}{\gtrless}} \frac{1}{B}\ln\frac{P_0}{P_1 A} = \zeta \tag{38}$$

III. PERFORMANCE OF THE OPTIMUM RECEIVER

In this chapter we will integrate the probability densities for X^2 that were derived in Section II-A to find expressions for the error probabilities P_M and P_F in terms of the threshold ζ. Then we will be able to obtain expressions for the ROC (P_M vs. P_F) and the total probability of error, $P(\mathcal{E})$. Finally, we will examine the phenomenon of "receiver guessing"—i.e. we will find that under certain conditions $P(\mathcal{E})$ will be smaller if we simply make a guess (based on the a priori knowledge) as to the signal's presence, without even looking at the correlation output X^2.

A. The Error Probabilities P_M and P_F, and the ROC.

Referring back to Figure 8, we see that to find P_M we must integrate the conditional density for X^2 given a signal is present from zero to ζ.

$$P_M = \int_0^{\zeta} p_{X^2}(Z|\text{signal})\,dZ \tag{39}$$

$$P_M = \int_0^{\zeta} \frac{1}{N_0 C} e^{-Z/N_0 C} dZ$$

$$P_M = 1 - e^{-\zeta/N_0 C} \tag{40}$$

Similarly,

$$P_F = \int_{\zeta}^{\infty} p_{X^2}(Z|\text{no signal})\,dZ \tag{41}$$

$$P_F = \int_{\zeta}^{\infty} \frac{1}{N_0} e^{-Z/N_0}\,dZ$$

$$P_F = e^{-\zeta/N_0} \tag{42}$$

To find an expression for the ROC, we simply eliminate ζ to give P_M as a function of P_F.

$$\ln P_F = -\zeta/N_0 \tag{43}$$

$$P_M = 1 - e^{(1/C)\ln P_F} \tag{44}$$

$$P_M = 1 - P_F^{\,1/[2(E_r/N_0)R^2(\tau) + 1]} \tag{45}$$

B. <u>The Total Probability of Error.</u>

If P_1 is the a priori probability that the signal is present, and P_0 is the a priori probability that it is absent, then

$$P(\mathcal{E}) = P_1 \cdot P_M + P_0 \cdot P_F \tag{46}$$

We found in Section II-C that the threshold which minimizes $P(\mathcal{E})$ is

$$\zeta = \frac{1}{B}\ln\frac{P_0}{P_1 A}$$

$$P_F = e^{-\zeta/N_0}$$

$$= \left(\frac{P_0}{P_1 A}\right)^{-\frac{1}{BN_0}} \tag{47}$$

Or defining

$$D = 1/BN_0 = 1 + \frac{1}{2(E_r/N_0)} \tag{48}$$

gives

$$P_F = \left(\frac{P_0}{P_1 A}\right)^{-D} \tag{49}$$

Similarly,

$$P_M = 1 - e^{-\zeta/N_0 C}$$

$$= 1 - \left(\frac{P_0}{P_1 A}\right)^{-1/CBN_0} \tag{50}$$

Or, using the same definition for D,

$$P_M = 1 - \left(\frac{P_0}{P_1 A}\right)^{-D/C} \tag{51}$$

Substituting (49) and (51) into (46) gives:

$$P(\varepsilon) = P_0\left(\frac{P_0}{P_1A}\right)^{-D} + P_1 - P_1\left(\frac{P_0}{P_1A}\right)^{-D/C} \tag{52}$$

C. Receiver Guessing.

Note that the threshold ζ is a function of both the SNR and the a priori probabilities P_0 and P_1. Hence, as these parameters change, so to will the desired threshold vary between the limits of zero and infinity. However, if we look back at our expression for the threshold (Eq. 38), we see that it becomes negative when P_0/P_1A becomes less than one. A negative threshold is, of course, physically impossible. The explanation for this lies in the fact that when P_0/P_1A is less than one, our decision rule is no longer optimum. Indeed, in this case we can do better (i.e. achieve a lower $P(\varepsilon)$) simply by making a guess based on P_0 and P_1, and ignoring the "data" (i.e. X^2). Take, for example, the extreme case of $P_1 = 1$ (and $P_0 = 1 - P_1 = 0$), i.e. we are certain that the signal is present without even looking at the correlation receiver, and the probability of error is therefore zero. However, if we set $P_0 = 0$ and $P_1 = 1$ in Equation 52, this results in $P(\varepsilon) = 1$. Clearly, we will do better by simply "guessing" (i.e. saying that the signal is present without looking at X^2).

We account for this phenomenon by modifying the decision rule as follows. When $P_0/P_1A \geq 1$, we base our decision on the threshold of Equation 38 as before (the threshold reaches zero when $P_0/P_1A = 1$). But when $P_0/P_1A < 1$, we base our decision entirely on the a priori knowledge.

Our final expression for the total probability of error is therefore:

$$P(\mathcal{E}) = \begin{cases} P_0\left(\dfrac{P_0}{P_1A}\right)^{-D} + P_1 - P_1\left(\dfrac{P_0}{P_1A}\right)^{-D/C} ; & \dfrac{P_0}{P_1A} \geq 0 \quad (53a) \\ P_0 = 1 - P_1 ; & \dfrac{P_0}{P_1A} < 0 \quad (53b) \end{cases}$$

with

$$D = 1 + 1/2(E_r/N_0)$$

$$C = 2(E_r/N_0)R^2(\tau) + 1$$

$$A = 1/[2(E_r/N_0) + 1]$$

Since $P_0 = 1 - P_1$, the point at which we begin guessing depends on two parameters: P_1 and the SNR (E_r/N_0). We guess when

$$\frac{P_0}{P_1A} = \frac{1 - P_1}{P_1A} < 1 \qquad (54)$$

$$\frac{1 - P_1}{P_1} \cdot [2(E_r/N_0) + 1] < 1 \qquad (55)$$

$$2P_1 + 2P_1(E_r/N_0) > 2(E_r/N_0) + 1$$

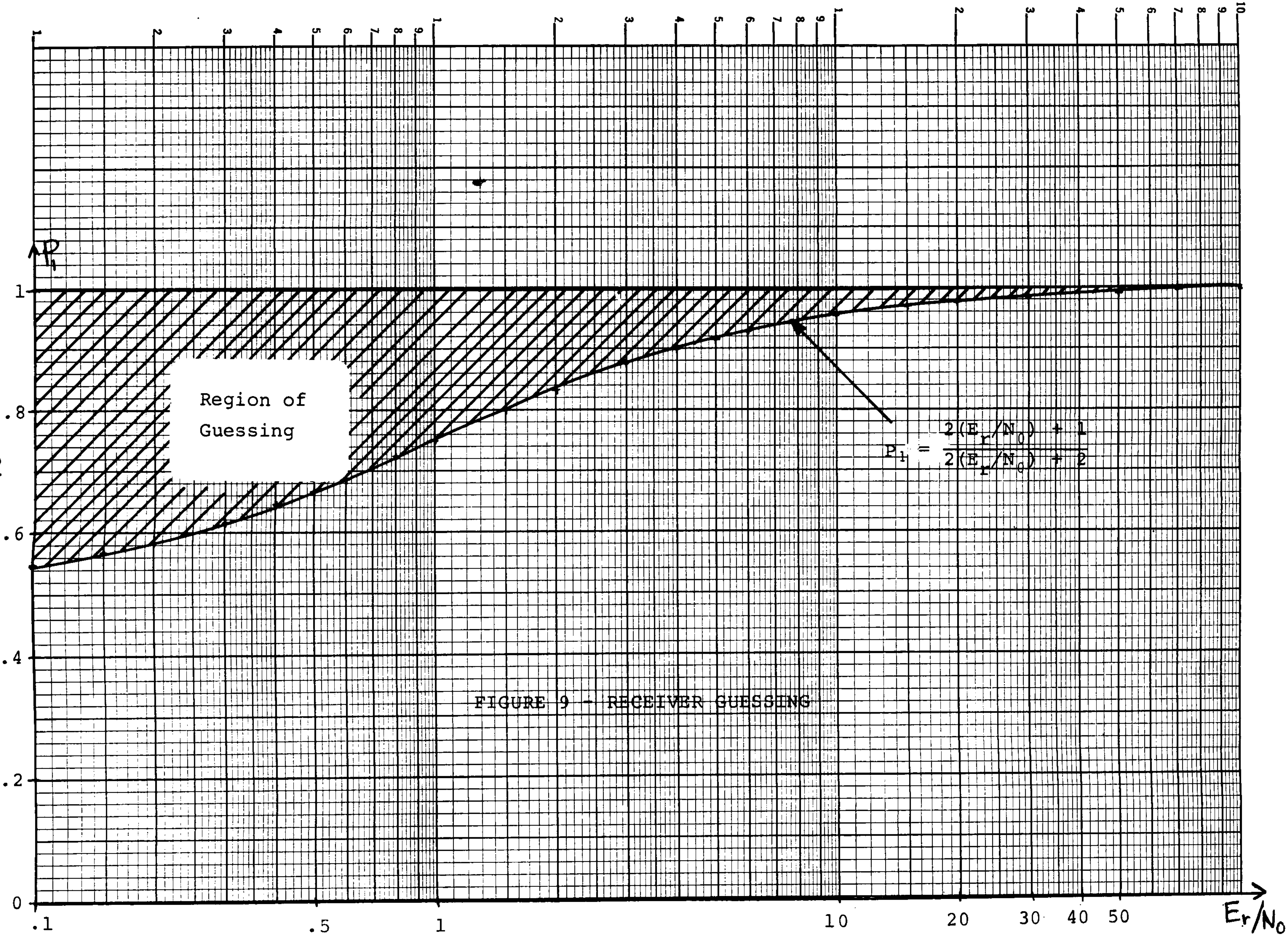


FIGURE 9 - RECEIVER GUESSING

$$P_1 > \frac{2(E_r/N_0) + 1}{2(E_r/N_0) + 2} \tag{56}$$

Equation 56 is plotted in Figure 9. The shaded region indicates those values for P_1 and the SNR for which guessing would improve the performance.

IV. DISPERSION

The reception scheme that has been described in the first three chapters does not take into account the dispersive effect of the plasma. The received signal will, in fact, be a corrupted version of the signal that was expected when the optimum receiver was designed, and the performance of the receiver will be degraded. Also, it is difficult to compensate for the effect before hand by changing the receiver design, since the degree of dispersion will change with time and will often be unpredictable.

The dispersion problem, particularly as it applies to Sunblazer, has been examined fairly thoroughly in Ref. 5. What follows in this chapter is simply an outline of that work, stressing, of course, its results.

Suppose now that our received signal is given not by Equation 2 but rather is of the form

$$r(t) = a\sqrt{2E_t}\,\mathrm{Re}[d(t - \tau_0)e^{j(\omega_0 t + \theta)}] + n(t) \tag{57}$$

where d(t), the dispersed version of s(t), can be complex, i.e.

$$d(t) = d_r(t) + jd_i(t) \tag{58}$$

If we trace this through the correlation receiver of Figure 4, we find that our output statistic is now

$$\begin{aligned} X^2(\tau) &= \frac{a^2E_t}{2}R^*_{ds}(\tau - \tau_0)R_{ds}(\tau - \tau_0) \\ &= \frac{a^2E_t}{2}|R_{ds}(\tau - \tau_0)|^2 \end{aligned} \tag{59}$$

where R_{ds} is the cross-correlation function of the dispersed and undispersed signals.

Note that the statistics of the situation have not changed. Dispersion is an energy-conserving process. Receiving a dispersed signal is equivalent to receiving an undispersed signal but correlating it (in the receiver) against a dispersed signal. For this reason, then, to find the error probabilities for the dispersed case, we need only substitute $R_{ds}(\tau)$ for $R(\tau)$ in equation 26.

The problem, then, boils down to actually computing the cross-correlation function $R_{ds}(\tau)$. To do this we first need a convenient representation of the dispersed signal. This is obtained by expanding the phase of the signal, $\phi(\omega)$, in a Taylor series to three terms. Only the result is given here (for the derivation see Ref. 5, pps. 33-39):

$$d(t) = \frac{1-j}{2} e^{j[\omega_0 t - \phi(\omega_0)]} \int_{-\infty}^{\infty} s[t - \phi'(\omega_0) + \sqrt{\pi\phi''(\omega_0)}\,u] e^{j\frac{\pi}{2}u^2} du \tag{60}$$

where ω_0 is the center frequency. This equation is based on several assumptions, namely that $s(t)$ is quasi-monochromatic, the attenuation does not vary much with respect to ω, the Taylor expansion of $\phi(\omega)$ exists and higher order terms in the expansion can be neglected, and finally the assumption that

$$\sqrt{\frac{\phi''(\omega_0)}{\pi}}\,\omega_0 >> 1; \text{ i.e. large dispersion.}$$

All of these assumptions were tested (Ref. 5, pps. 39-46) and found to hold for the Sunblazer experiment.

Note that the amount of dispersion is related to $\phi''(\omega_0)$. A good measure of the amount of dispersion is the "dispersion time" given by

$$\tau_0 = \sqrt{\pi\phi''(\omega_0)} \tag{61}$$

If we now evaluate the cross-correlation function we find it to be

$$R_{ds}(\tau) = \frac{1 + j}{2}e^{-j\theta'} \int_{-\infty}^{\infty} R_{ss}(\tau - \tau_0 u)e^{-j\frac{\pi}{2}u^2} du \tag{62}$$

We are only interested in the magnitude of this function, namely,

$$|R_{ds}(\tau)| = \{(\mathrm{Re}[R_{ds}(\tau)])^2 + (\mathrm{Im}[R_{ds}(\tau)])^2\}^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left\{\left[\int_{-\infty}^{\infty} R_{ss}(\tau - \tau_0 u)\cos\frac{\pi}{2}u^2 du\right]^2 + \left[\int_{-\infty}^{\infty} R_{ss}(\tau - \tau_0 u)\sin\frac{\pi}{2}u^2 du\right]^2\right\}^{1/2} \tag{63}$$

Finally, if s(t) is a binary signal(e.g. a Barker Code) with baud length (elementary pulse width) T, then it can be shown (Ref. 5 pps. 48-50) that the degree of corruption caused by dispersion depends only on the dispersion ratio, $\beta = \tau_0/T$.

In other words, the shape (ignoring scaling) of $|R_{ds}(\tau)|$ depends only on the dispersion ratio β.

This function has been computed in the case of eleven-bit Barker Code for several different values of β (Ref. 5, ch IV).

V. RESULTS

This chapter presents the computational results for the numerical calculation of the ROC's and the total probability of error curves for the case of the eleven-bit Barker Code. The $P(\mathcal{E})$ curves are plotted as functions of the four system parameters β (dispersion ratio), τ (miss time), P (a priori probability of signal present), and SNR.

A. Computational Methods.

A good part of the computations were performed using the MAP (Mathematical Analysis Program) system through Compatible Time Sharing on the IBM 7094 computer. MAP replaces the normal procedures of programming with direct command-response interchange between the user and the computer, and makes available a large number of useful subroutines (e.g. for curve plotting). For a detailed description of the system, the reader is referred to the MAP manual (Ref. 3).

Two problems with MAP in its present form are the limited number of things that it can do (e.g. it cannot handle arrays of two or more dimensions), and the large amounts of computer time that it uses. It was therefore found necessary or advantageous to write programs in MAD, using MAP subroutines, for the computation of the different $P(\mathcal{E})$ curves, and then execute the programs within MAP using MAP commands.

B. Calculation of the ROC's.

The first MAD program in Appendix B calculates, for equation 45, the ROC's for different signal-to-noise ratio. In this case R = 1; i.e. the receiver pinpoints the main peak exactly and there is no dispersion. This set of curves gives the most general description of receiver performance for "optimum" conditions (i.e. for $\tau = 0$ and $\beta = 0$).

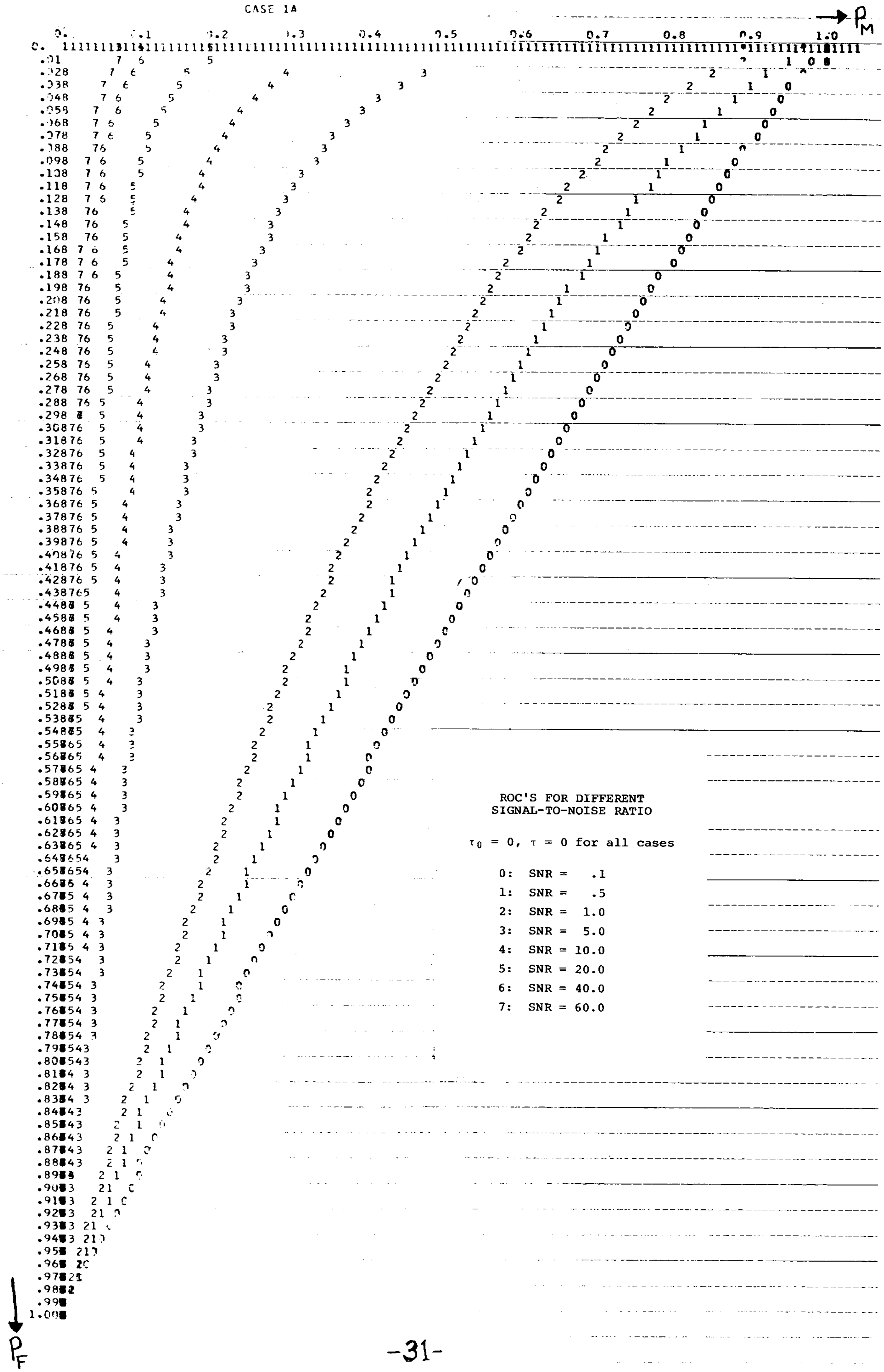
By fixing the SNR and reading in values of R as data cards, the program will compute and plot the ROC's for different values of β. The proper values of R for different β are computed by another MAD program (CORREL).

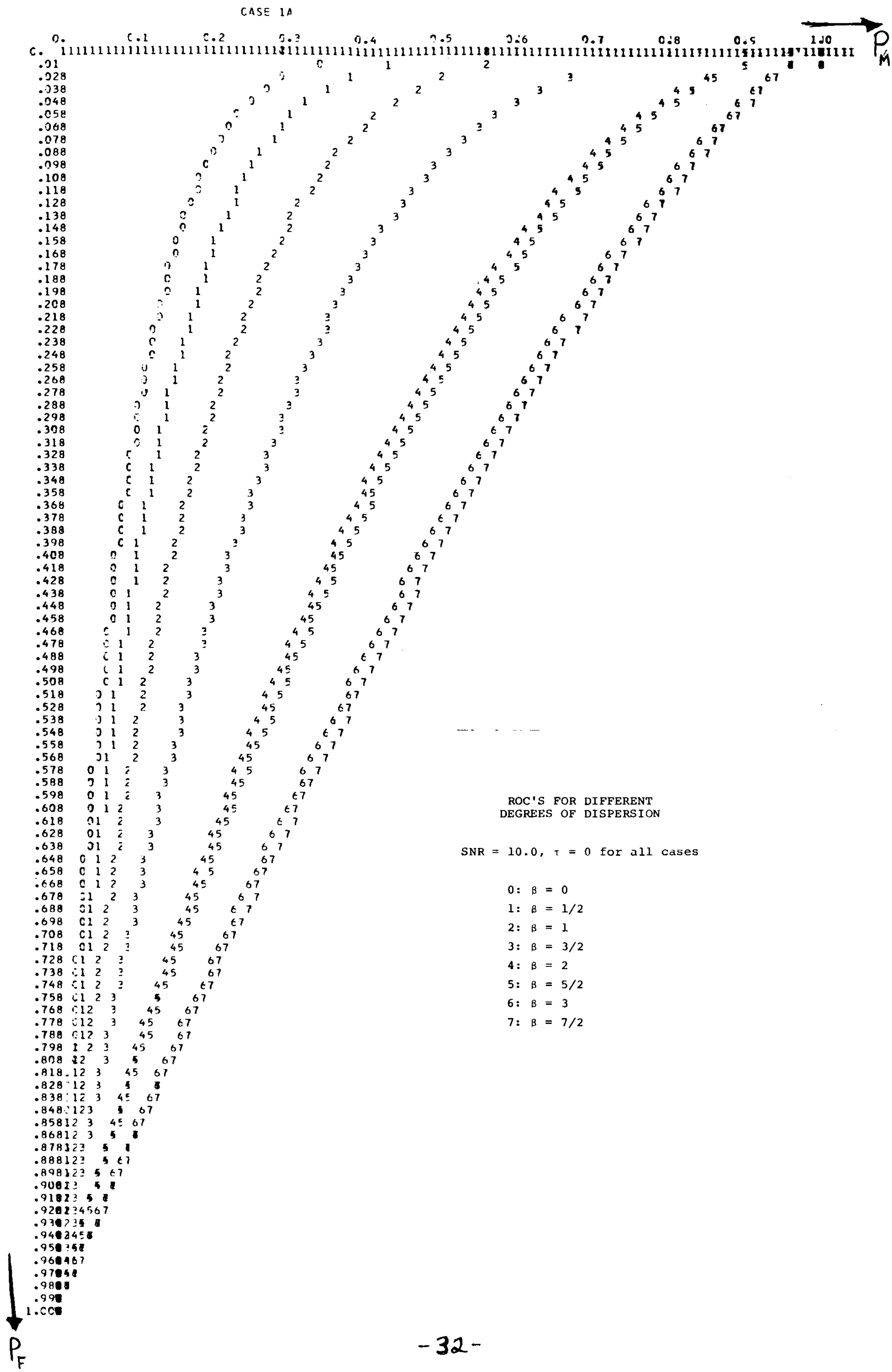
The ROC's appear on the next four pages. Observe that for low SNR (.2) performance is so poor that it hardly even matters how much dispersion there is.

C. Calculation of the Total Probability of Error Curves.

Although the ROC's provide the most general description of receiver performance, the total probability of error is often a more meaningful and understandable performance measure. For this reason an abundance of $P(\mathcal{E})$ curves have been computed as functions of the four system parameters described earlier.

All of the $P(\mathcal{E})$ curves come from Equation 53, hence they describe the performance of the receiver whose decision rule (threshold) was determined under the assumptions of no dispersion and an infinite number of correlators (resulting in zero miss time)—i.e. the assumption that R = 1. This, in

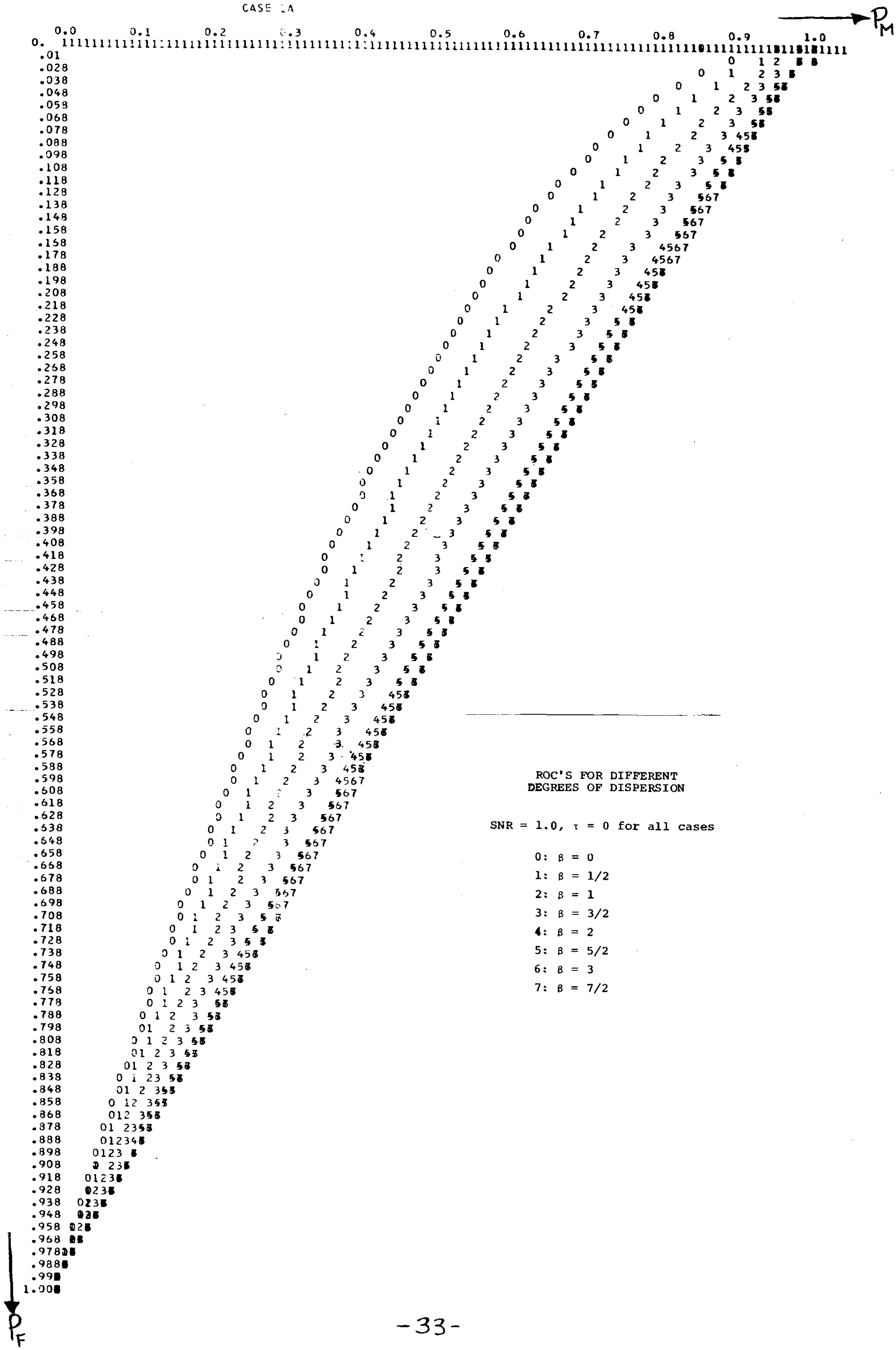

CASE 1A
P_M
P_F
ROC'S FOR DIFFERENT
SIGNAL-TO-NOISE RATIO
$\tau_0 = 0$, $\tau = 0$ for all cases
0: SNR = .1
1: SNR = .5
2: SNR = 1.0
3: SNR = 5.0
4: SNR = 10.0
5: SNR = 20.0
6: SNR = 40.0
7: SNR = 60.0



ROC'S FOR DIFFERENT
DEGREES OF DISPERSION

SNR = 10.0, τ = 0 for all cases

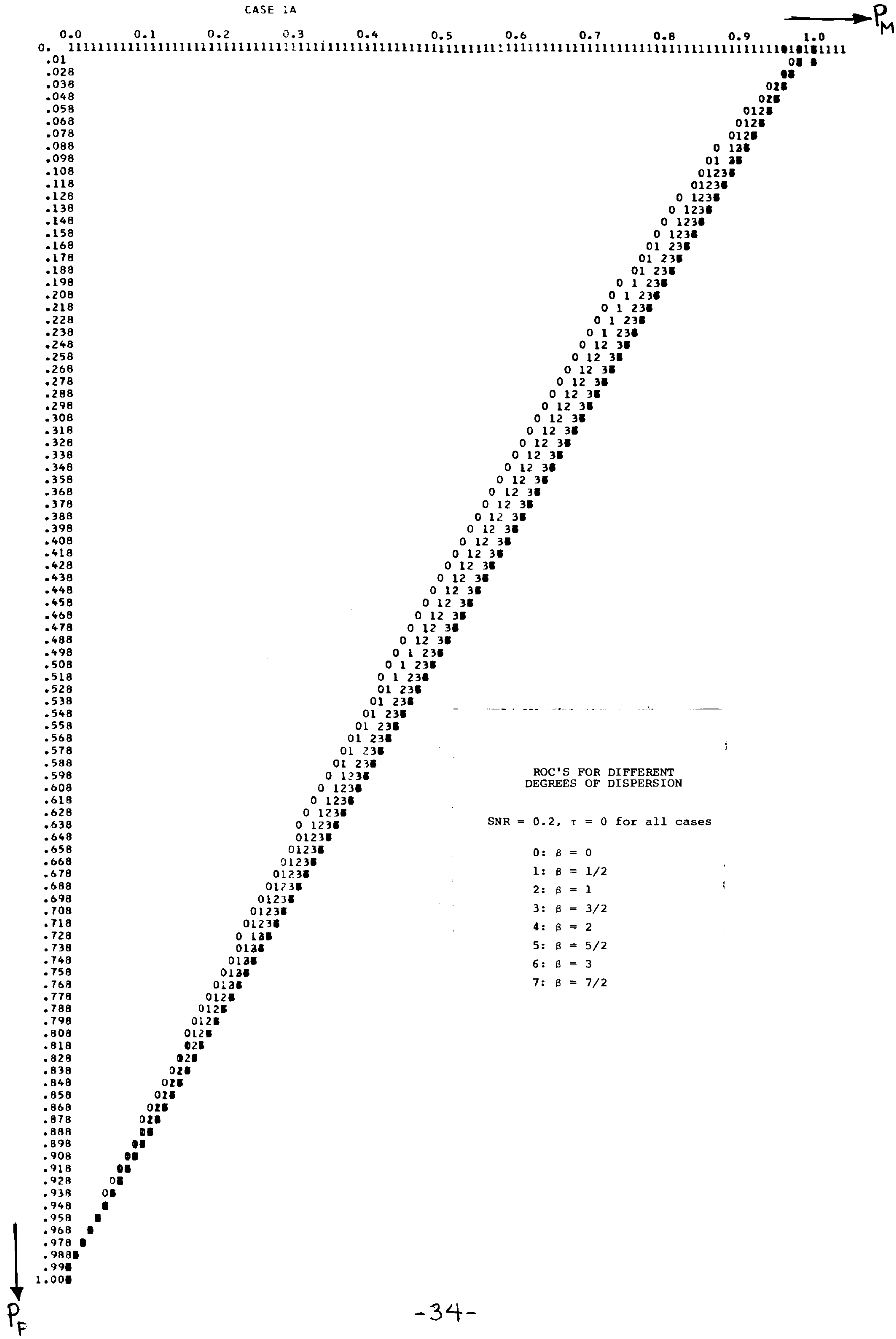
0: $\beta = 0$
1: $\beta = 1/2$
2: $\beta = 1$
3: $\beta = 3/2$
4: $\beta = 2$
5: $\beta = 5/2$
6: $\beta = 3$
7: $\beta = 7/2$



ROC'S FOR DIFFERENT
DEGREES OF DISPERSION

SNR = 1.0, τ = 0 for all cases

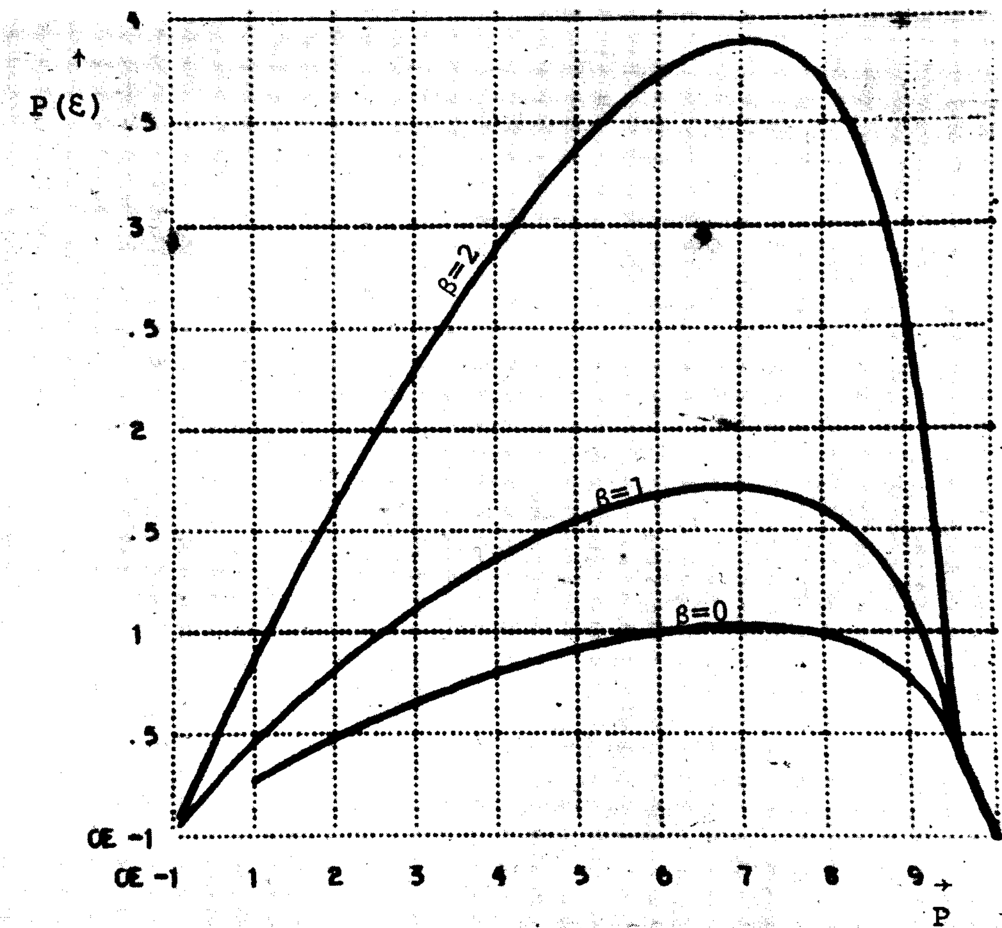
0: β = 0
1: β = 1/2
2: β = 1
3: β = 3/2
4: β = 2
5: β = 5/2
6: β = 3
7: β = 7/2


CASE 1A
P_M
P_F
ROC'S FOR DIFFERENT
DEGREES OF DISPERSION
SNR = 0.2, τ = 0 for all cases
0: β = 0
1: β = 1/2
2: β = 1
3: β = 3/2
4: β = 2
5: β = 5/2
6: β = 3
7: β = 7/2

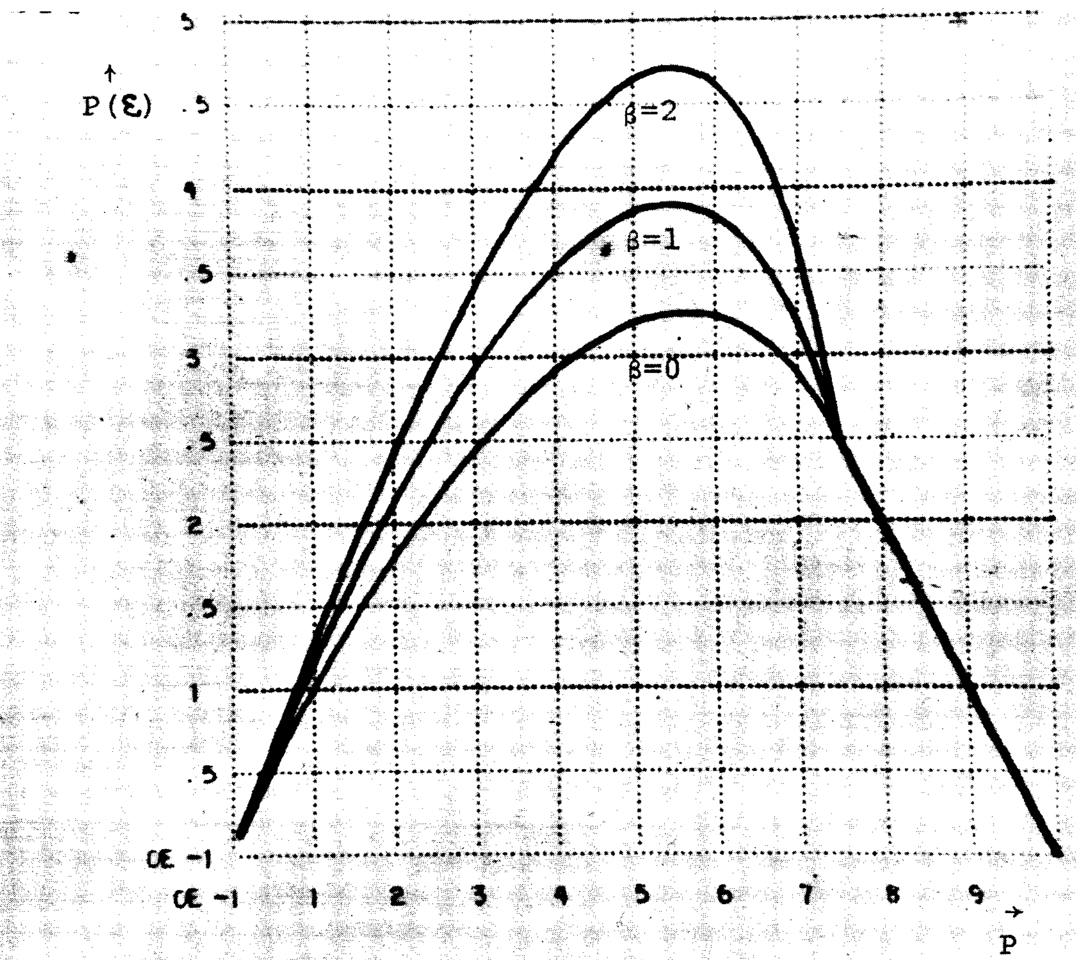
fact, was how the "optimum receiver" was defined (Chapter II). In effect, then, these curves show the sensitivity of the optimum receiver to dispersion and to a non-zero miss time. As is evident from the results which follow, receiver performance can be degraded quite severely by these effects, in fact sometimes to the point where the probability of error can be reduced by simply making a guess based on the a priori knowledge. In the next chapter we will discuss how the optimum receiver could be re-designed (i.e. how to pick a new threshold) if indeed <u>we knew beforehand</u> what the dispersion and miss time would be.

One parameter that can change drastically during the Sunblazer mission is P, the a priori probability that the signal is present. When the satellite is being tracked (which hopefully would be most of the time) P will be large (.9 or .95), but if the satellite is "lost" we would have no a priori knowledge and P would be .5.

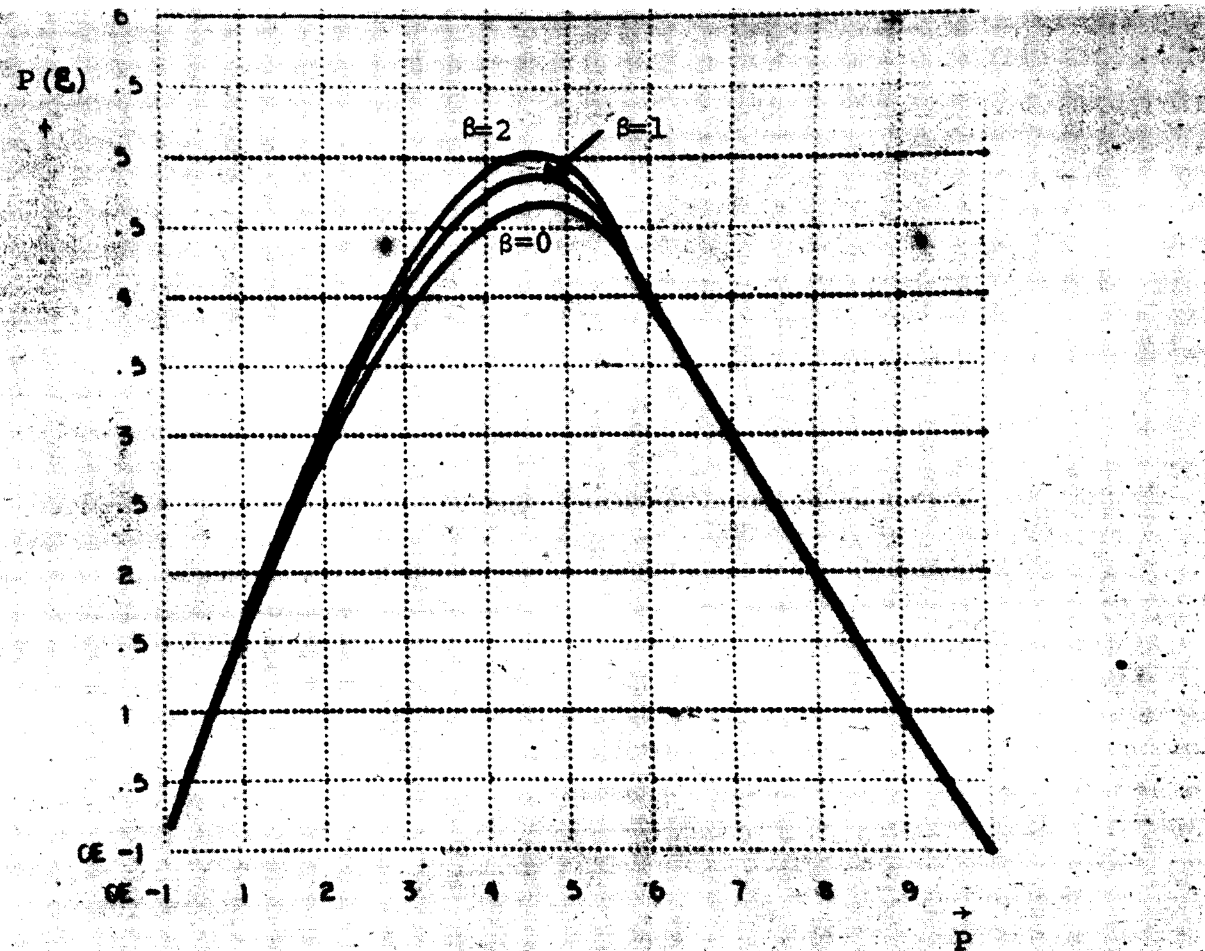
A MAD program was written (TPE—see Appendix) to calculate the total probability of error as a function of P for different amounts of dispersion and different signal-to-noise ratio. Plots were made using the MAP system and appear on the following pages. The vertical line segments on each of these plots represents regions of receiver guessing—i.e. points for which $(1 - P)/PA < 1$, the threshold has reached zero, and we automatically say that the signal is present. But note that if the line segment is extended upward, and the dispersion is large, it runs <u>below</u> part of the $P(\mathcal{E})$ curve. Again, this is because the threshold was picked under the assumption there would be no



TOTAL PROBABILITY OF ERROR VS A PRIORI PROBABILITY THAT SIGNAL IS PRESENT: SNR = 10, τ = 0



P(ε) VS P: SNR = 1, τ = 0



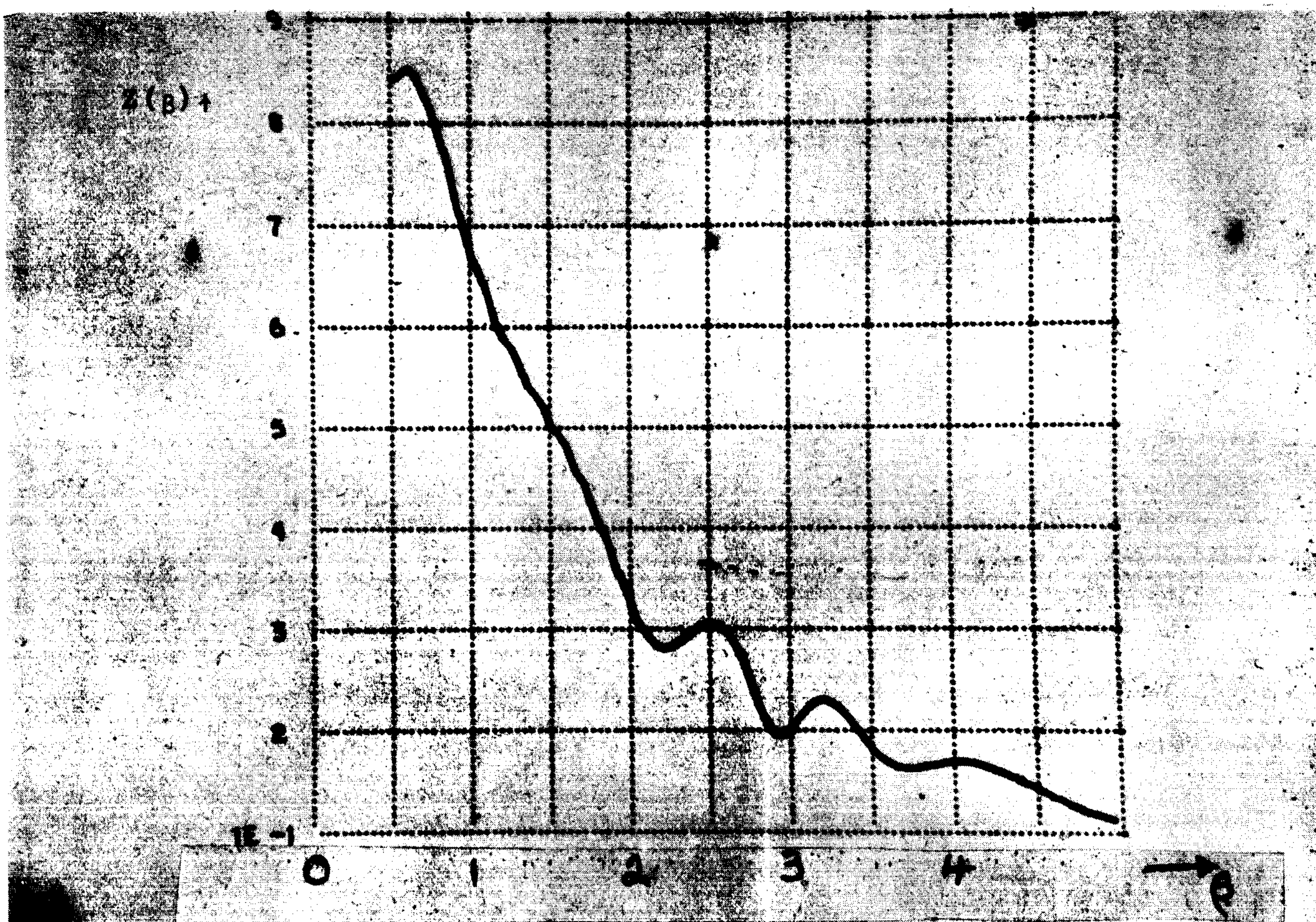
P(E) VS P: SNR = .2, τ = 0

dispersion, and hence is no longer optimum when dispersion is introduced. Note also that the peaks of these curves (especially for large SNR) fall not at P = .5 but at some larger value of P. This should not be surprising, however, if one remembers that P_M and P_F have quite different probability densities that are not symmetric by themselves, and certainly not symmetric with respect to each other. A plot of $P(\mathcal{E})$ is, really, just a Bayesian Risk curve, and there is no reason for a Bayesian Risk curve to be symmetric if its composite densities are different.

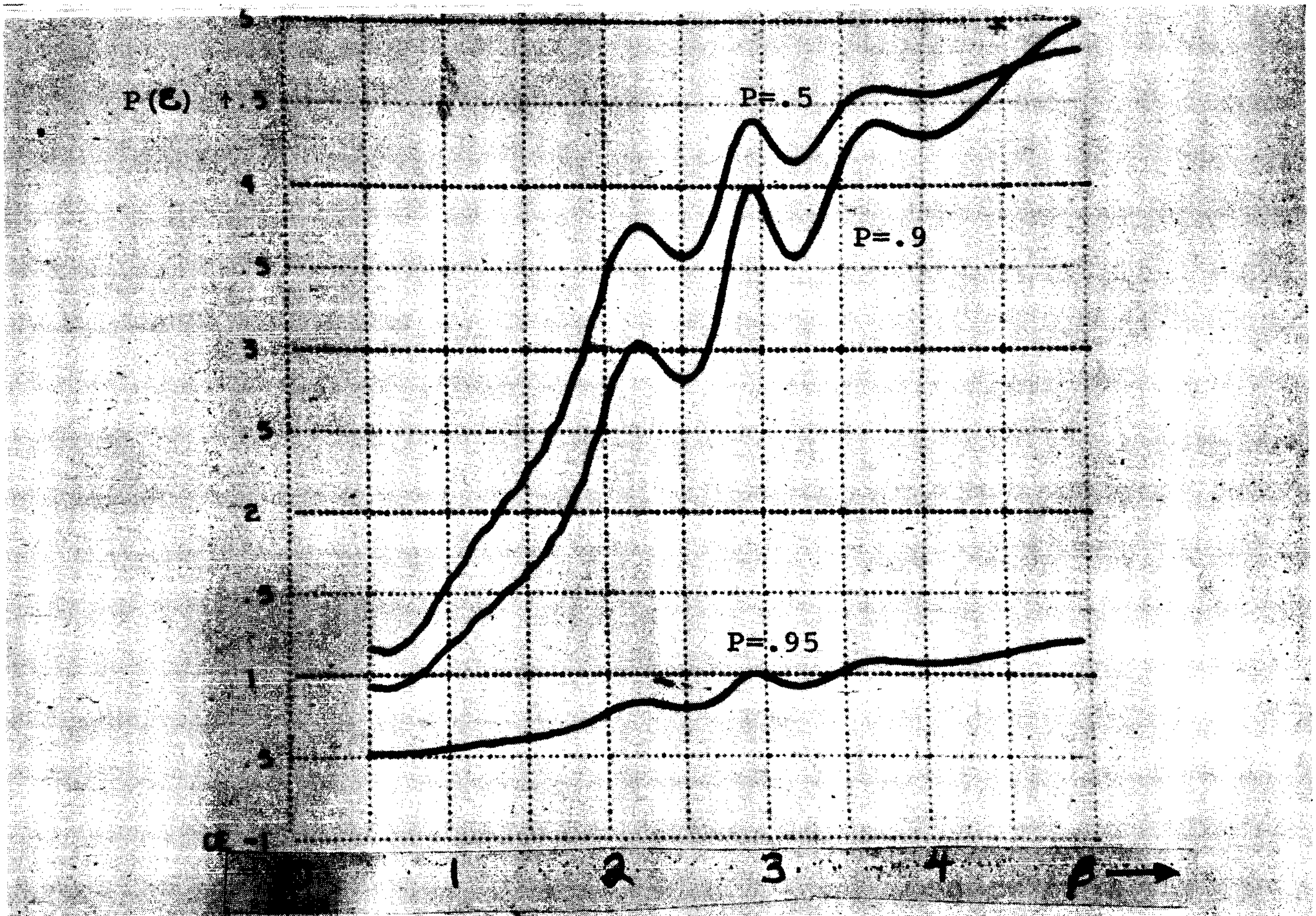
To calculate $P(\mathcal{E})$ as a function of the dispersion ratio β, it is first necessary to calculate $Z(\beta)$, the peak correlator output as a function of β. The program to do this (CORREL) was written using Equation 63, and a plot of the results follow. Note that this curve is not monotonically decreasing. The reason for this is that the autocorrelation function of an eleven-bit Barker Code has side lobes which are negative. Hence, as the dispersion increases and the individual lobes spread out further and further, the side lobes will subtract from the main lobe in a periodic fashion.

To go from $Z(\beta)$ to a plot of $P(\mathcal{E})$ as a function of β could be done using only MAP commands. The results of those calculations follow.

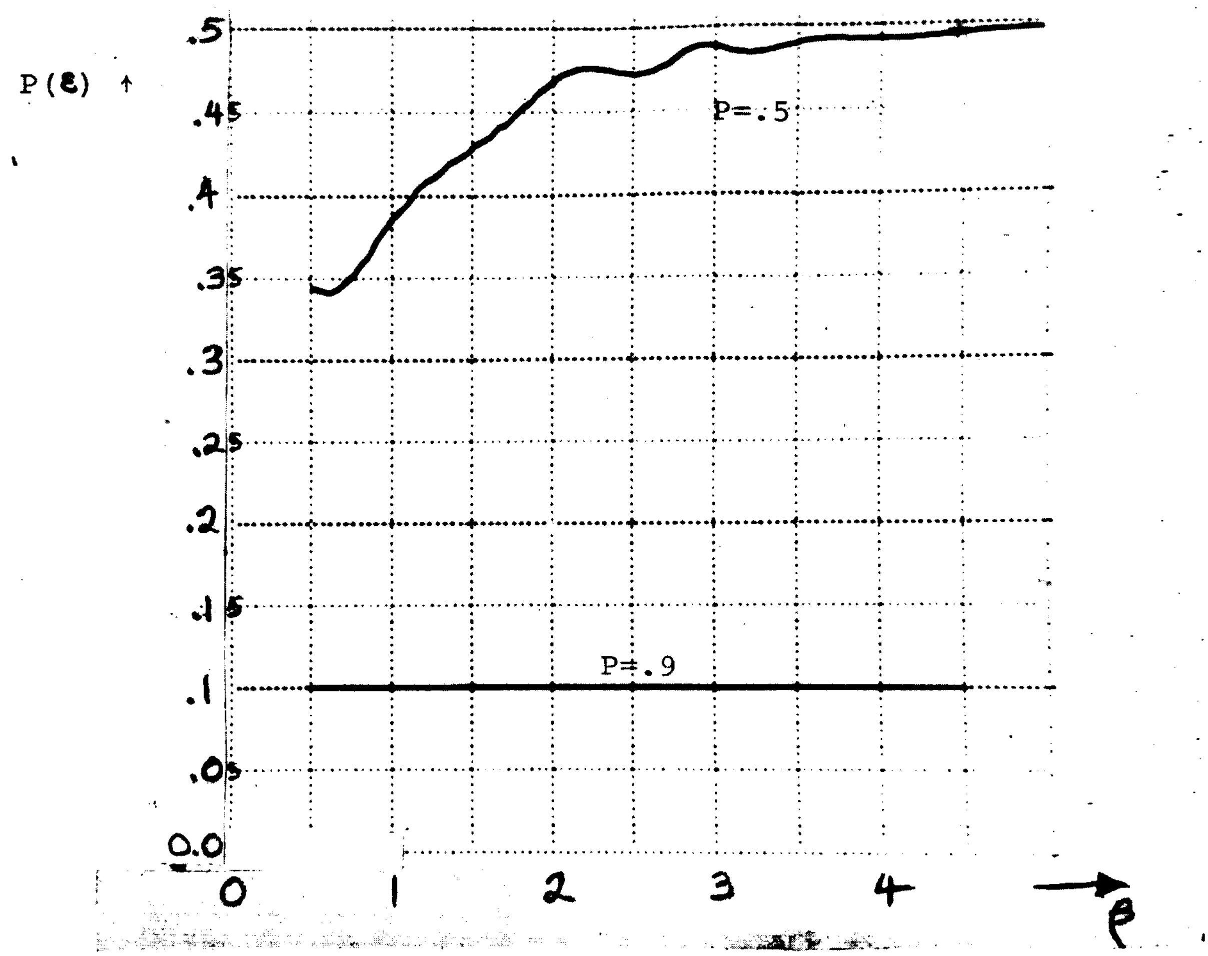
The plots of $P(\mathcal{E})$ as a function of SNR were calculated (PESNR) for different values of β, and for the three cases P = .5, .9, and .95. Note, for example, how, in the case of P = .9, when the horizontal line segment represting receiver



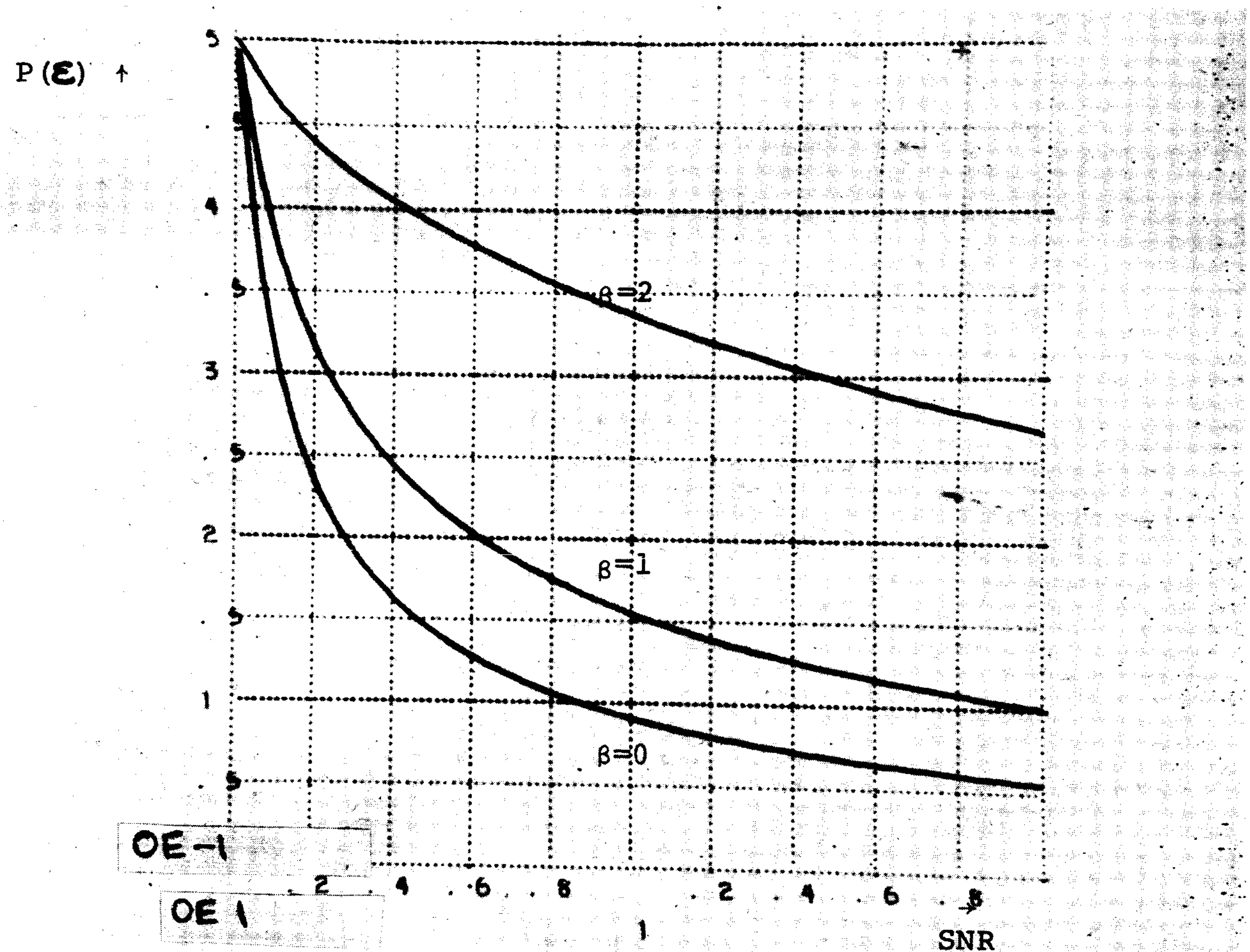
PEAK CORRELATION OUTPUT (NORMALIZED) VS DISPERSION RATIO β



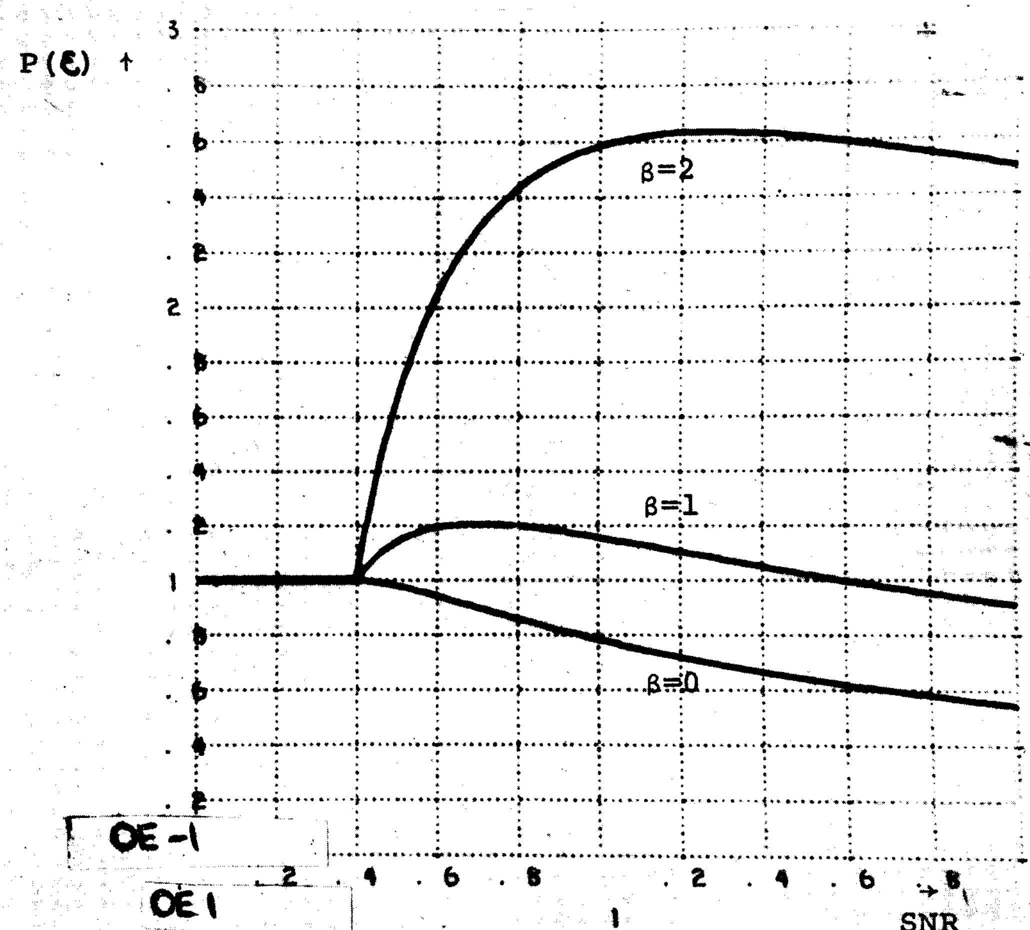
TOTAL PROBABILITY OF ERROR VS DISPERSION RATIO β:
SNR = 10, τ = 0



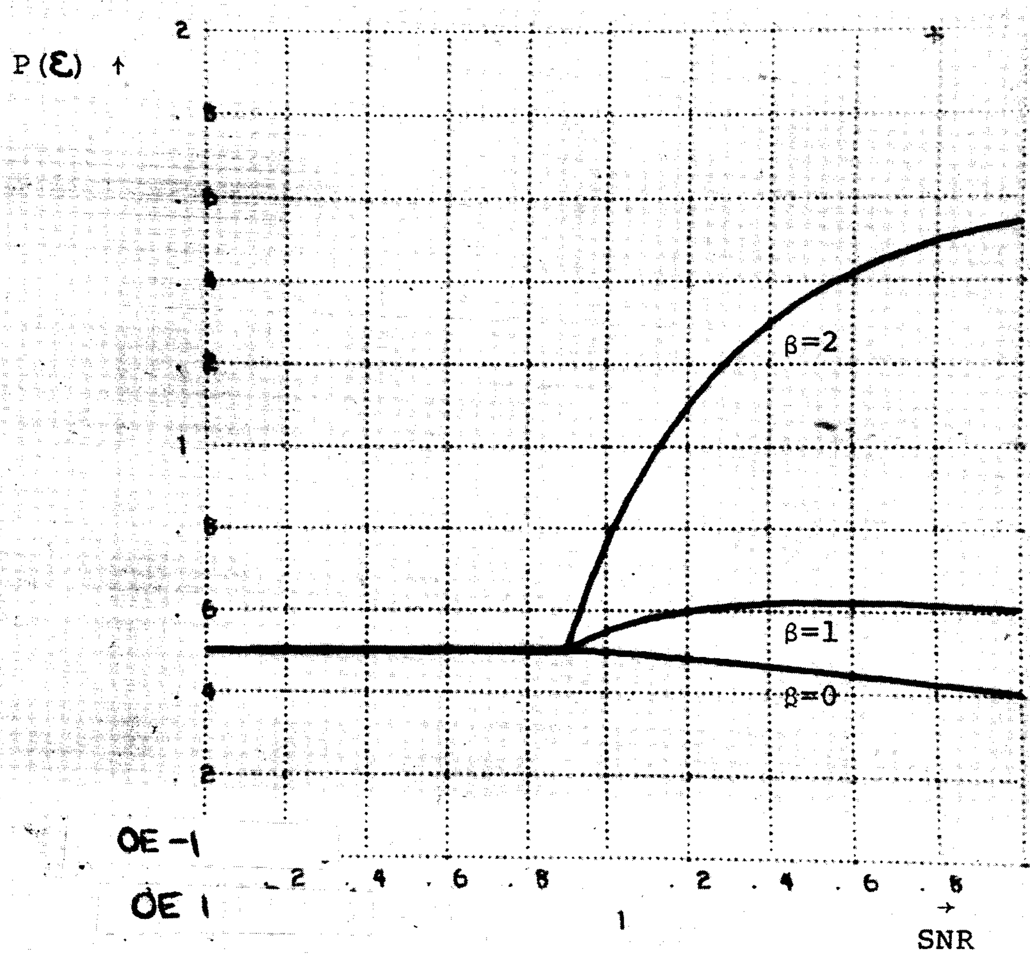
P(ε) VS β: SNR = 1, τ = 0



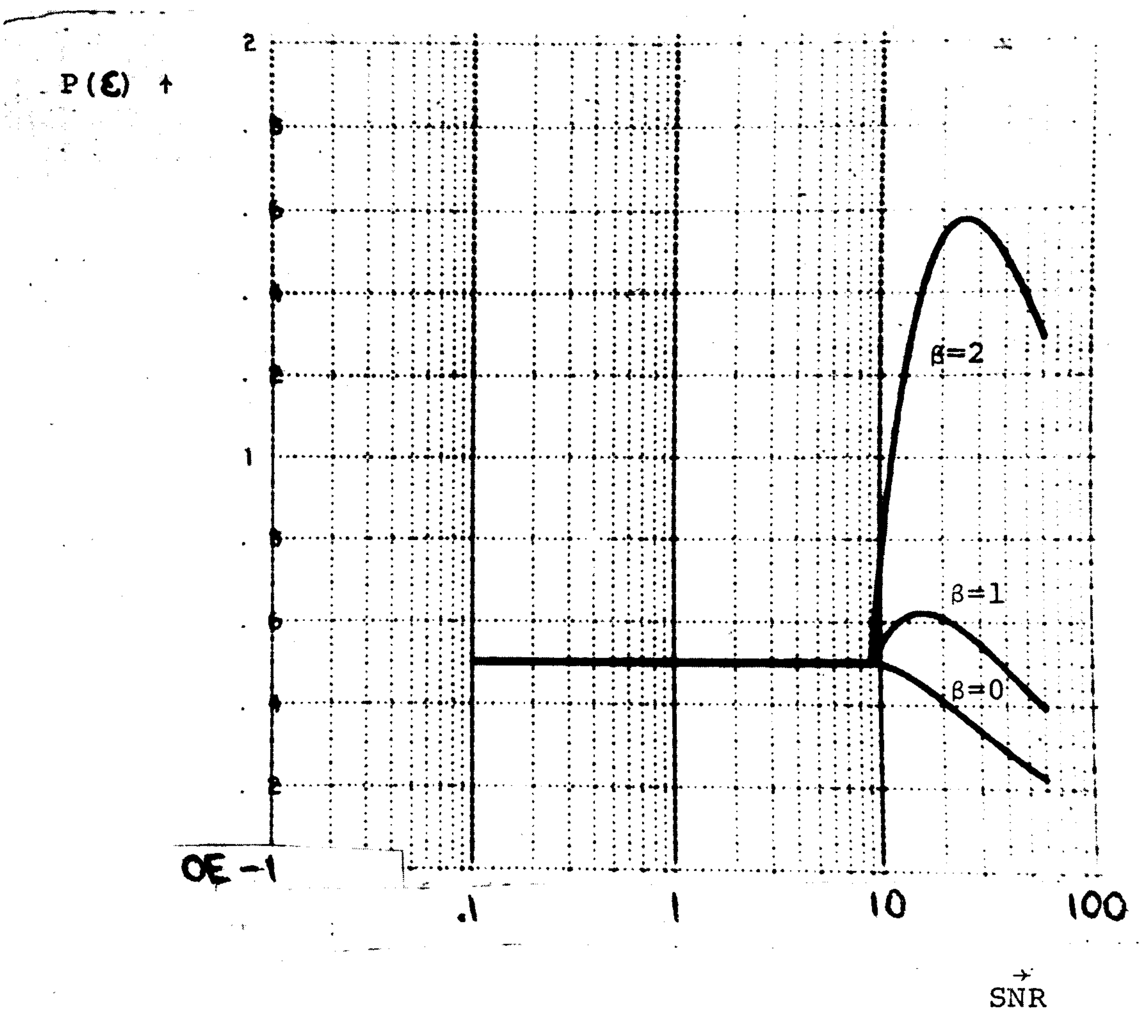
TOTAL PROBABILITY OF ERROR VS SIGNAL-TO-NOISE RATIO:
P = .5, τ = 0



P(ε) VS SNR: P = .9, τ = 0



P(Ɛ) VS SNR: P = .95, τ = 0

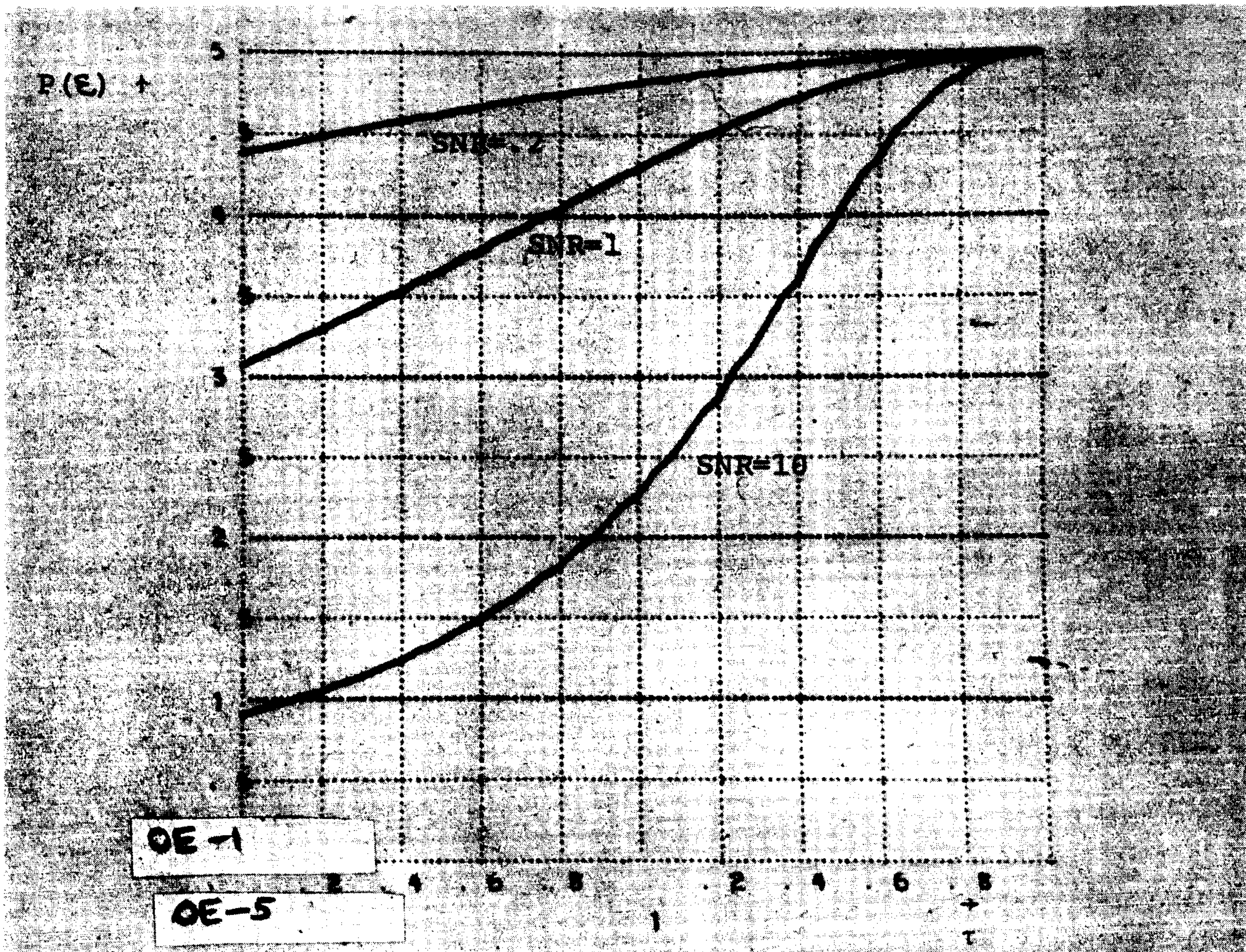


P(ℰ) VS SNR: P = .95, τ = 0

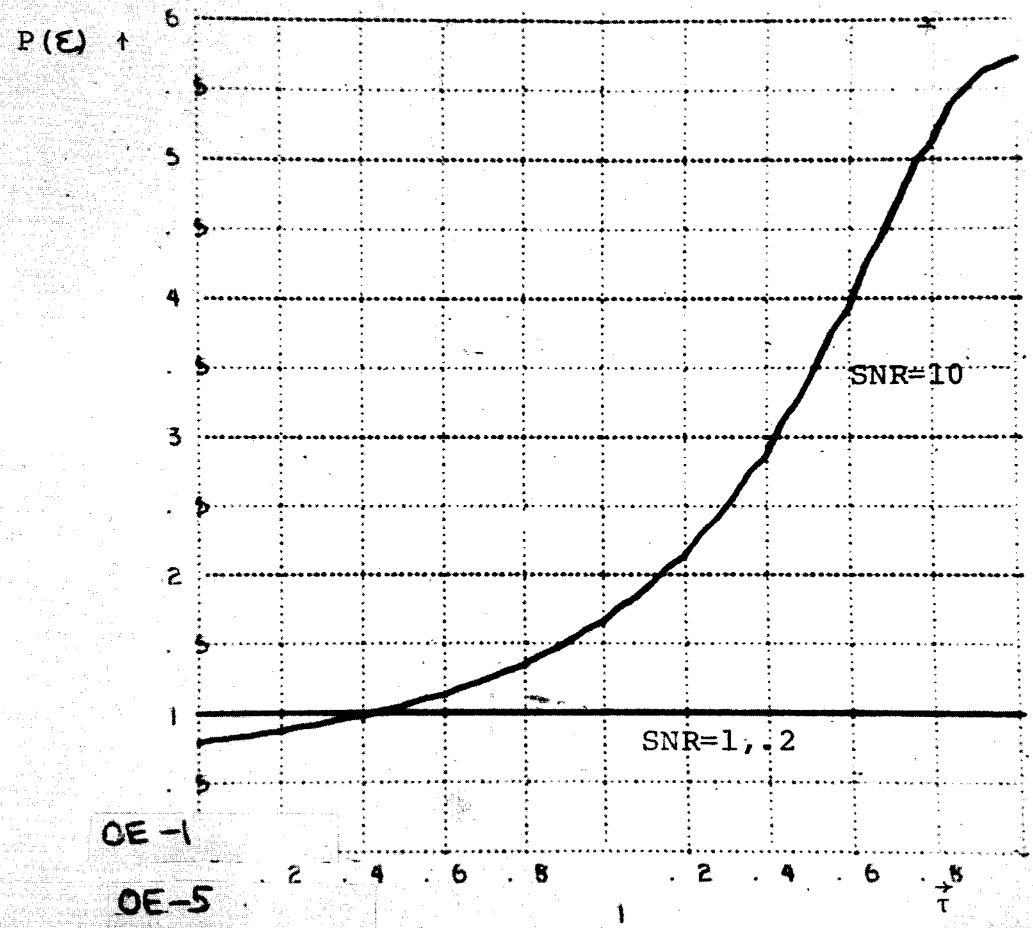
guessing is extended to the right it falls well below the P($\mathcal{E}$) curves for non-zero dispersion. The case of P = .95 was re-plotted on a semi-log graph with a larger range of SNR to show how as the SNR increases, the P($\mathcal{E}$) for the dispersed case eventually slopes back downward to fall below the line for guessing.

For the case of no dispersion, plots of P($\mathcal{E}$) as a function of the miss time τ could be calculated directly from the autocorrelation function of the Barker Code (with MAD program PETT). For non-zero dispersion it was necessary first to calculate the cross-correlation function of the dispersed and undispersed Barker Code (pulse width of 20μsec) from Equation 63 (MAD program CRSCOR; also see Ref. 5). Then a second MAD program (PETTY) could be executed to obtain the desired result of P($\mathcal{E}$) vs. τ.

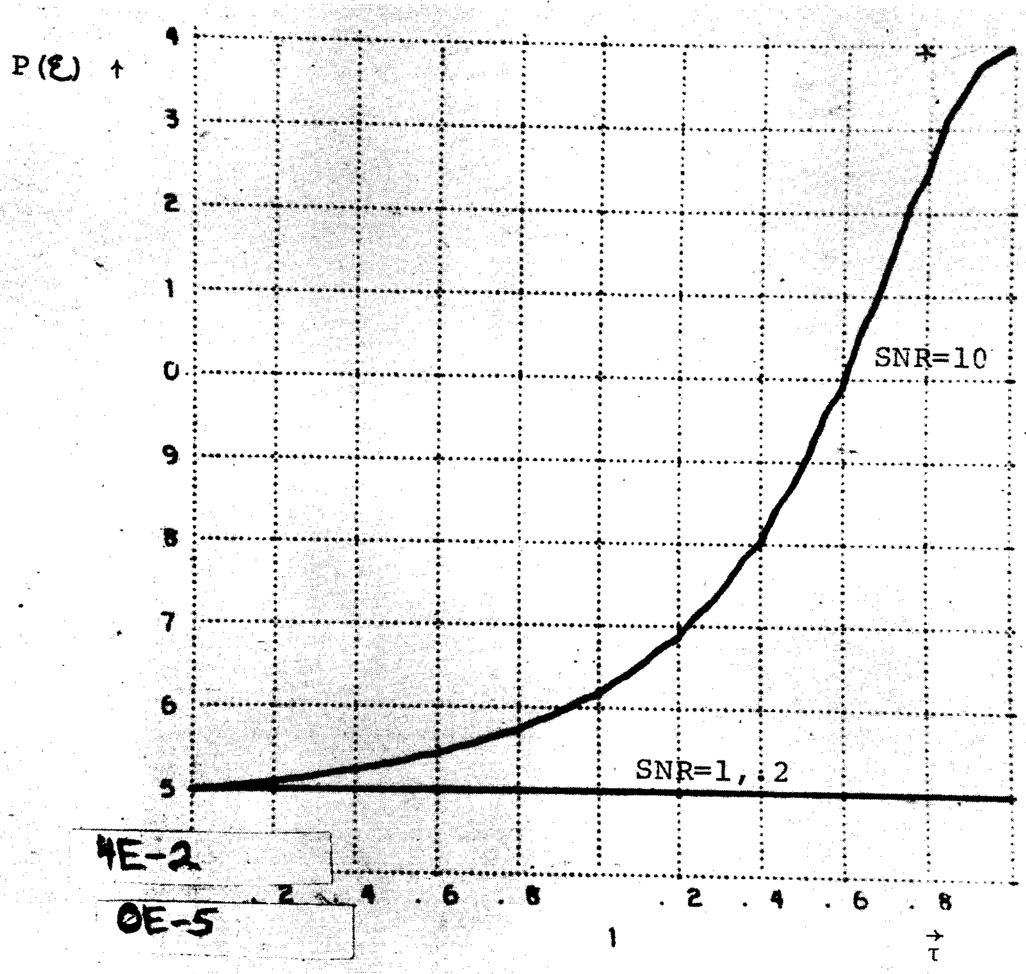
Again, note that whenever the a priori knowledge is high (i.e. P = .9 or .95), the resulting P($\mathcal{E}$) is larger than the P($\mathcal{E}$) that would be obtained by guessing (which is 1-P). Also, note that in the dispersed cases, the curve for $\beta = 1$ may fall below the curve for $\beta = 0$ (as τ increases) and then rise above the curve for $\beta = 2$. The reason for this is simply that the cross-correlation function changes its shape quite drastically as the dispersion increases. For certain values of τ the cross-correlation function may be larger for $\beta = 1$ then for $\beta = 0$; in fact this is likely to be the case since as β increases the function becomes less highly peaked—i.e. the main peak spreads out. In effect then, when the dispersion



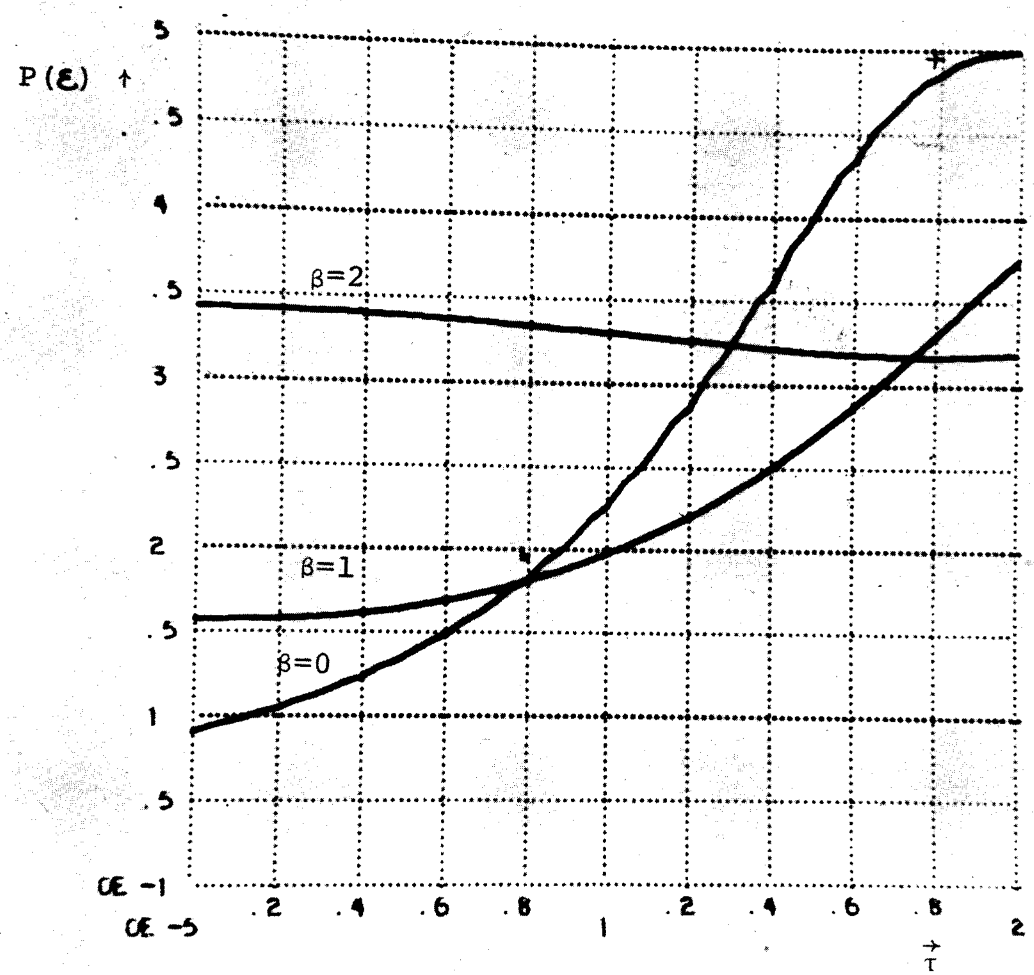
TOTAL PROBABILITY OF ERROR VS MISS TIME τ: β = 0,
P = .5 (BAUD LENGTH = 20μsec



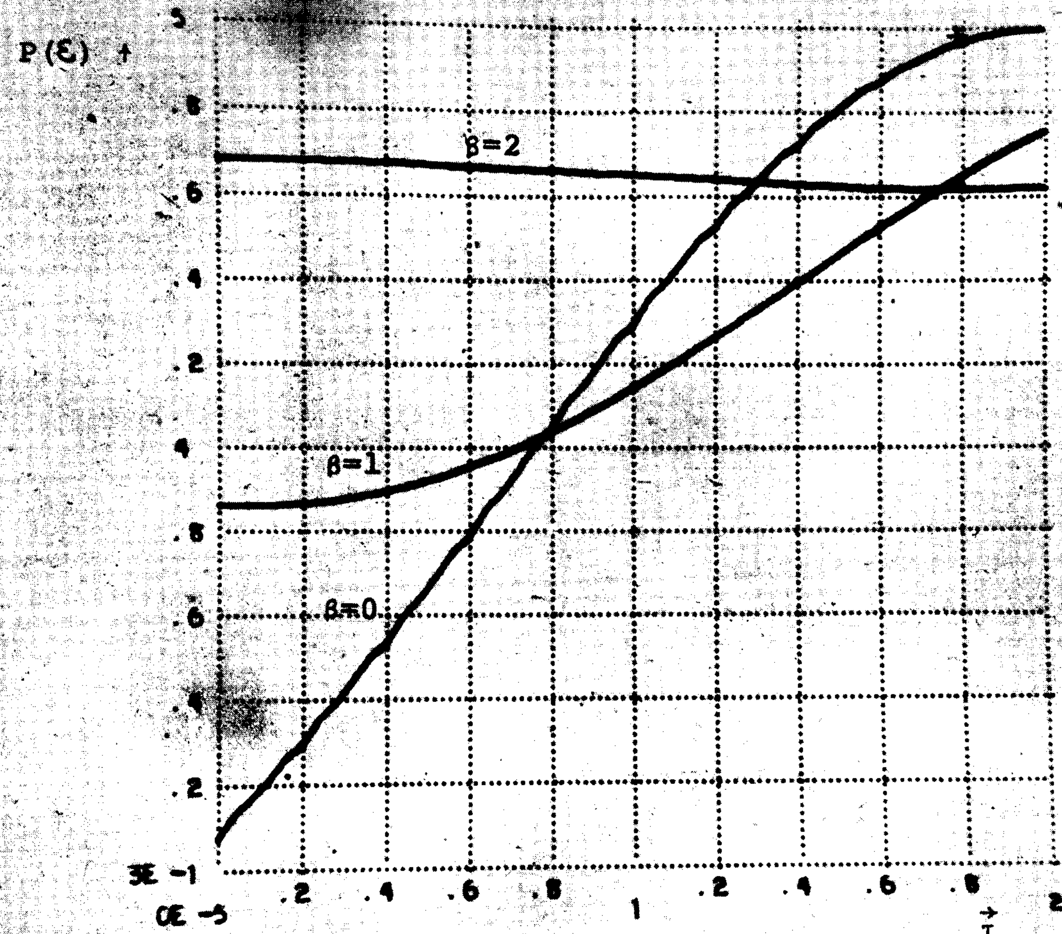
P(Ɛ) VS τ: β =0, P̄ = .9



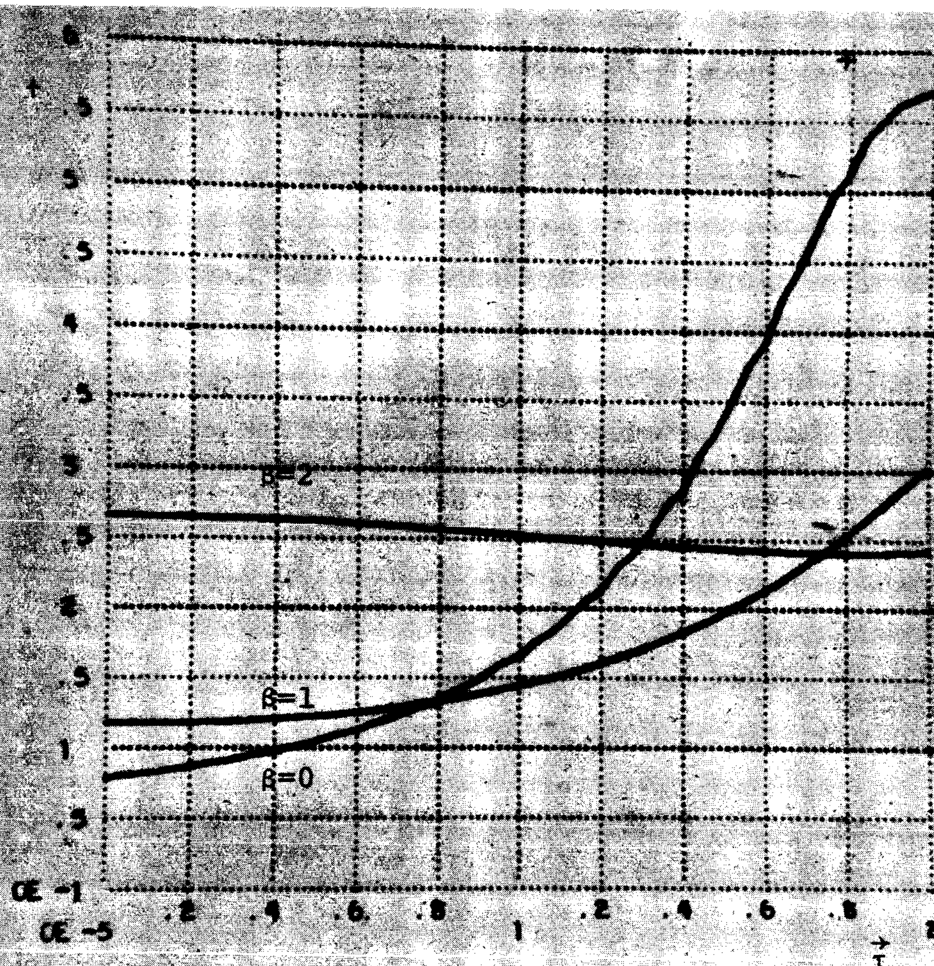
P(ε) VS τ: β =0, P = .95



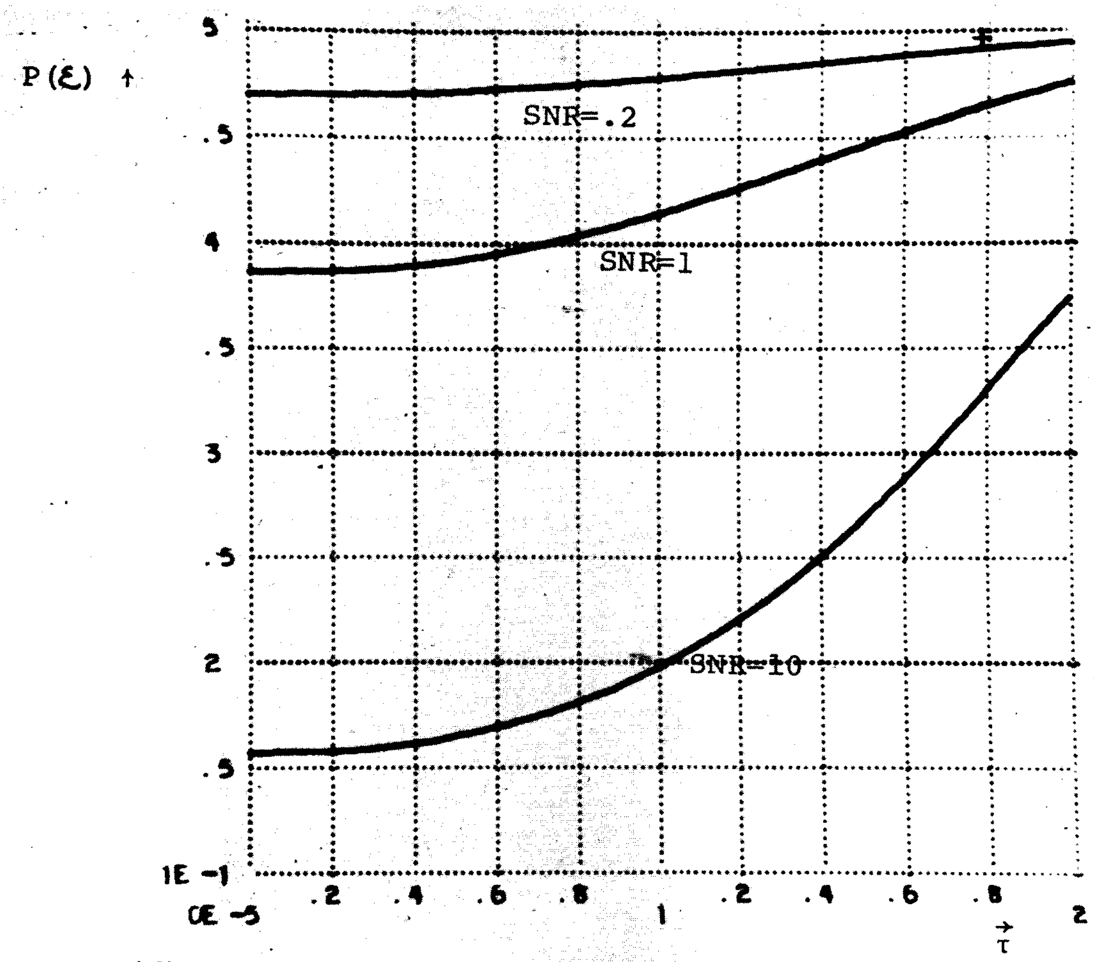
P(ε) VS τ: SNR = 10, P = .5



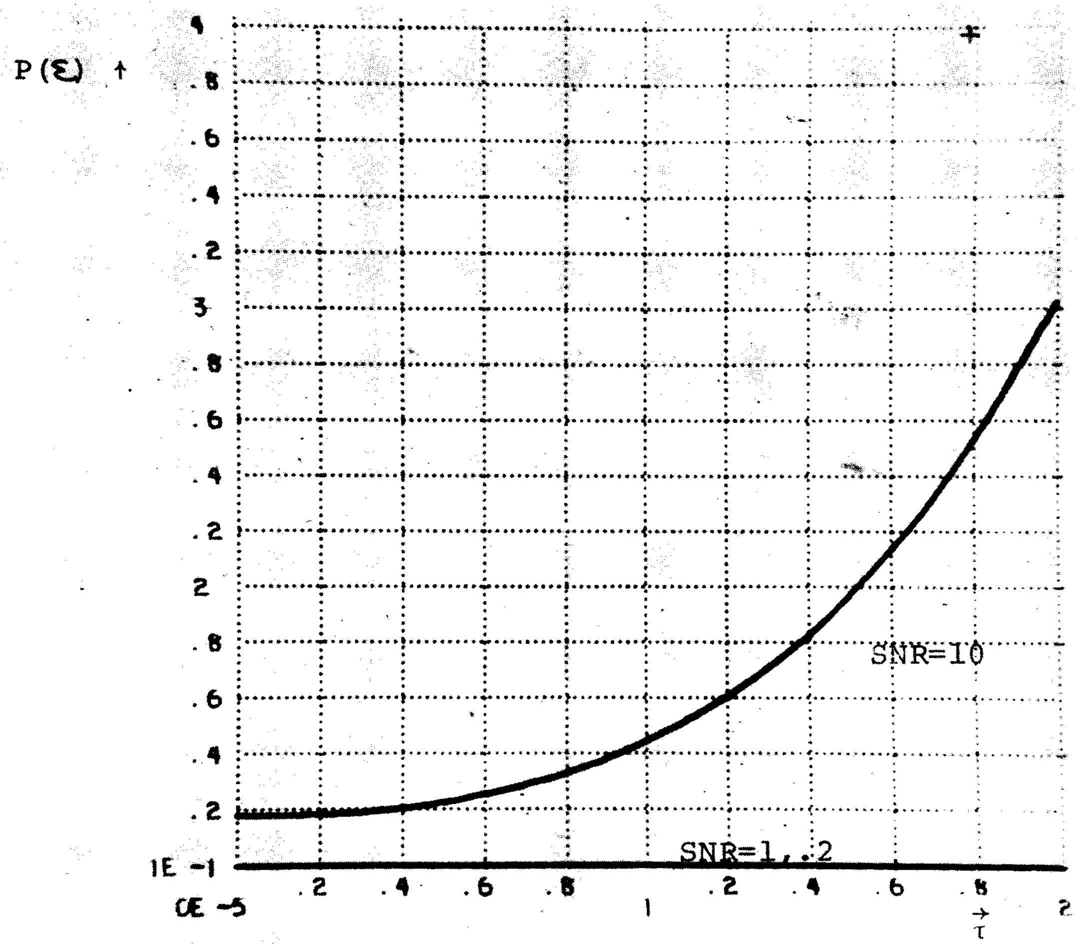
P(ℰ) VS τ: SNR = 1, P = .5



P(ε) VS τ: SNR = 10, P = .9



P(ℰ) VS τ: β = 1, P = .5



P(ε) VS τ: β = 1, P = .9

is large the receiver will be less sensitive to a large miss time. Hence, if the SNR is high, and there are not many correlators in the receiver, it can even be advantageous for the medium to be dispersive. Of course we would certainly expect to have enough correlators so that this would never be the case; nonetheless, the phenomenon is interesting in itself.

VI. CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

The results of the previous chapter showed to a good degree of accuracy how, for an eleven-bit Barker Code, the performance of the optimum receiver is affected by changing four parameters of the system. We found that for high dispersion or miss time, i.e. for cases when $R(\tau)$ was significantly less than unity, the receiver did not in fact perform optimally. That is, we found that the total probability of error was higher than it would have been if we had "guessed", i.e. if we had automatically said that the signal was present without even looking at the correlation output. This was particularly true if the a priori probability that the signal is present was high (and hence the $P(\mathcal{E})$ resulting from guessing was low).

These results are due, of course, to the fact that we designed the receiver (picked the threshold) under the assumption of zero dispersion and zero miss time, i.e. under the assumption that $R(\tau) = 1$ (which is how we went from Equation 32 to Equation 37). Let us assume for the moment that we could know ahead of time what the dispersion and miss time would be. It is then easy to see how we could revise the threshold so that it would again be optimum in minimizing the total probability of error. If we set $C_{00} = C_{11} = 0$ and $C_{10} = C_{01} = 1$ in Equation 32, we would have for the threshold:

$$\zeta = \frac{N_0 C}{C - 1} \ln\left[\frac{C(1 - P)}{P}\right] \qquad (64)$$

with

$$C = 2\frac{E_r}{N_0}R^2(\tau) + 1$$

as in Equation 34.

As $C(1 - P)/P$ became smaller (approaching one) the threshold ζ would approach zero. Finally, for $C(1 - P)/P \leq 1$, the threshold would remain at zero and we would always say that the signal is present (i.e. "guess"), thus giving a total probability of error of $1 - P$. Of course this region of receiver guessing would be different than it was before. Whereas before the guessing region depended only on P and the SNR (as in Figure 9), it now depends also on β and τ. Remember that we want to guess whenever

$$(1 - P)\left(\frac{1 - P}{PA}\right)^{-D} + P - P\left(\frac{1 - P}{PA}\right)^{-D/C} \geq 1 - P \tag{65}$$

which occurs whenever

$$C(1 - P)/P \leq 1 \tag{66}$$

or

$$(1 - P)\,2\frac{E_r}{N_0}\,R^2(\tau) + (1 - P) \leq P$$

or

$$R^2(\beta,\tau) \leq \frac{(2P - 1)}{2\,\frac{E_r}{N_0}\,(1 - P)} \tag{67}$$

$R^2(\tau)$, however, is usually a rather complicated function of β and τ. Nonetheless, assuming that we could write R^2 analytically in terms of β and τ, we could determine the region in 4-space (coordinates are P, SNR, τ, β) in which the threshold would be zero and we would guess. Since the threshold would now be determined by Equation 64, the receiver would again be optimum.

With a large enough number of correlators, τ can be made close enough to zero so as to be negligible. For a fixed pulse width and carrier frequency, however, β is beyond our control. Furthermore, β is generally somewhat unpredictable, particularly when it is large enough to be significant. For this reason Equation 64 is rather limited in usefulness, and we would still pick the threshold as before from Equation 38.

Note that we have studied the receiver performance for a range of SNR, β, and τ that depict rather bad operating conditions. We would hope that for the actual Sunblazer mission τ would be very small, and that at least most of the time β would be small and the SNR would be large. These conditions would yield rather small probabilities of error, which of course is what we desire. However these conditions, as desirable as they are, are not very interesting to look at. It is more interesting, and more rewarding, to study the receiver performance as it will be when conditions are at their worst (e.g. around the time of conjunction). This was the reason for choosing the values of the system parameters that we did.

This work, although complete in itself, does suggest other problems worthy of investigation. The first of these is a performance analysis, along the lines of the one presented in this paper, for the two signal and noise case. We would like to get more information from the Sunblazer experiment than just the electron density of the extended corona. For example, there is a certain amount of telemetry data that we would like to transmit, as well as the possibility of other on-board experiments. One reasonable way of doing this is to send one of two possible signals (e.g. a forward and backward Barker Code) which would be relatively orthogonal to each other. There are now three possible "messages" to choose from—no signal, signal 1, and signal 2. A performance analysis would certainly be in order for this scheme.

This entire communication scheme depends on the assumption that the phase of the signal will remain relatively constant over its duration. Since there is now good reason to suspect that this will not be the case, it is likely that a better communication scheme for Sunblazer is to transmit an amplitude-modulated signal, and receive it via envelope detection. The performance of this scheme will also depend on such system parameters as P, τ, β, and SNR. Exactly how it depends on these parameters should certainly be determined.

VII. APPENDICES

A. Another Derivation of the Minimum $P(\mathcal{E})$ Threshold.

In Chapter II we found the threshold that minimized the total probability of error by evaluating the likelihood-ratio test. We can get the same result simply by writing $P(\mathcal{E})$ as a function of ζ, differentiating with respect to ζ, and setting equal to zero.

$$P(\mathcal{E}) = P_1 P_M + P_0 P_F$$

Using equations 40 and 42,

$$P(\mathcal{E}) = P_1 - P_1 e^{-\zeta/N_0 C} + P_0 e^{-\zeta/N_0}$$

$$\frac{\partial P(\mathcal{E})}{\partial \zeta} = \frac{P_1}{N_0 C} e^{-\zeta/N_0 C} - \frac{P_1}{N_0} e^{-\zeta/N_0} = 0$$

$$\frac{P_1}{P_0 C} = e^{-\zeta/N_0 + \zeta/N_0 C} = e^{\zeta\left(\frac{1-C}{N_0 C}\right)}$$

$$\zeta = \frac{N_0 C}{C - 1} \ln \frac{P_0 C}{P_1}$$

Again, when $R = 1$, then $C = 1/A$ and $\frac{N_0 C}{C - 1} = 1/B$, and

$$\zeta = \frac{1}{B} \ln \frac{P_0}{P_1 A}$$

as before (Eq. 38).

B. Computer Programs.

The following pages contain the MAD programs used to compute the ROC's and $P(\mathcal{E})$ curves of Chapter V. The last page contains a sample command-response exchange in MAP for the calculation of some $P(\mathcal{E})$ curves.

MAD PROGRAM LISTING ... ... (Calculation of ROC's)

```
          REQUIRES DATA CARD FOR R

           DIMENSION Y(200),PF(2000,AD),PM(2000,AD)
           VECTOR VALUES AD=2,1,10
           INTEGER J,K,I
           FORMAT VARIABLES Y,J
           R=1.
           THROUGH DAT,FOR I=0,1,I.G.7
           READ DATA
           PRINT FORMAT LABEL,I
           VECTOR VALUES LABEL=$1H1,T30,2HPF,T60,2HPM,T90,I2/*$
           EXP=1/(SN*R*R+1)
           THROUGH CALC, FOR K=0,1,K.G.100
           PF(K,I)=.01*K
           PM(K,I)=1-PF(K,I).P.EXP
           PRINT FORMAT LIST, PF(K,I),PM(K,I)
           VECTOR VALUES LIST=$T25,F10.8,T55,F10.8*$
CALC       CONTINUE
DAT        CONTINUE
           PRINT COMMENT $1                                     CASB 1A$
           PRINT FORMAT GRAPH
           VECTOR VALUES GRAPH = $1H0,T9,3H0.0,T19,3H0.1,T29,3H0.2,T39,3
         0 H0.3,T49,3H0.4,T59,3H0.5,T69,3H0.6,T79,3H0.7,T89,3H0.8,T99,3H
         1 0.9,T109,3H1.0*$
           PRINT COMMENT $           11111111111111111111111111111111111
         0 111111111111111111111111111111111111111111111111111111111111
         1 1111111$
           THROUGH DO, FOR K=0,1,K.G.100
           THROUGH GONE, FOR J=0,1,J.G.I
           Y(J)=100.0*PM(K,J)
           WHENEVER Y(J).G.100 .OR.Y(J).L.0
               PRINT COMMENT $     DATA POINT IS NOT WITHIN GRAPH'S RANG
         0 E $
               TRANSFER TO GONE
           OR WHENEVER Y(J).L.1.0
               PRINT FORMAT ZZZ,J
               VECTOR VALUES ZZZ=$1H+,T10,I1*$
               TRANSFER TO GONE
           END OF CONDITIONAL
           PRINT FORMAT KEY,J
           VECTOR VALUES KEY=$1H+,T10,S'Y(J)',I1*$
GONE       CONTINUE
           Z=.01*K
           PRINT FORMAT BOTM,Z
DO         CONTINUE
           VECTOR VALUES BOTM=$1H+,F8.2/1H/*$
           END OF PROGRAM
```

```
   TPE     MAD       07/28  1418.7

              DIMENSION PE(200),AB(200)
              INTEGER K,MINP,MAXP
              EXECUTE RANGE.($P*$,MINP,MAXP,DELP)
              SN=VALUE.($SN*$)
              Z=VALUE.($Z*$)
              D=1+1/2*SN
              A=1/(2*SN+1)
              C=2*SN*Z*Z+1
              THROUGH ALPHA, FOR K=MINP,1,K.G.MAXP
              P=DELP*K
              AB(K-MINP)=P*A/(1-P)
              WHENEVER AB(K-MINP).GE.1., TRANSFER TO BETA
              PE(K-MINP)=(1-P)*AB(K-MINP).P.D-P*AB(K-MINP).P.(D/C)+P
              TRANSFER TO ALPHA
BETA          CONTINUE
              PE(K-MINP)=1-P
              TRANSFER TO ALPHA
ALPHA         CONTINUE
              PRINT RESULTS SN,Z
              EXECUTE OUT.($PE(P)*$,PE,MINP,MAXP,DELP)
              EXECUTE CHNCOM.
              END OF PROGRAM
R 1.083+.300
```

```
 CORREL    MAD       07/12  1523.2

              DIMENSION R(1000),Y(1000),C(1000),S(1000)
              INTEGER MINV,MAXV,J,I,MINT,MAXT
              EXECUTE IN.($R(V)*$,R,MINV,MAXV,DELV)
              EXECUTE RANGE.($T*$,MINT,MAXT,DELT)
              THROUGH SAM,FOR J=MINT,1,J.G.MAXT
              T=J*DELT
              TOS=T.P.2
              INTEGA=0
              INTEGB=0
              THROUGH CALC,FOR I=MINV,1,I.G.MAXV
              V=DELV*I
              ARG=1.5708*(V).P.2/TOS
              INTEGA=INTEGA+R(I-MINV)*COS.(ARG)
              INTEGB=INTEGB+R(I-MINV)*SIN.(ARG)
 CALC         CONTINUE
              Y(J-MINT)=.70711*SQRT.(DELV.P.2*(INTEGA.P.2+INTEGB.P.2)/TOS)
 SAM          CONTINUE
              EXECUTE OUT.($Y(T)*$,Y,MINT,MAXT,DELT)
              EXECUTE CHNCOM.
              END OF PROGRAM
 R 1.166+.400
```

```
 PESNR     MAD       07/28  1441.4

          DIMENSION PB(200),AB(200),C(200),D(200)
          INTEGER K,MINSN,MAXSN
          EXECUTE RANGE.($SN*$,MINSN,MAXSN,DELSN)
          Z=VALUE.($Z*$)
          P=VALUE.($P*$)
          THROUGH ALPHA,FOR K=MINSN,1,K.G.MAXSN
          SN=DELSN*K
          AB(K-MINSN)=P/((2*SN+1)*(1-P))
          WHENEVER AB(K-MINSN).GE. 1.0,TRANSFER TO BETA
          C(K-MINSN)=2*SN*Z*Z+1
          D(K-MINSN)=1+1/2*SN
          PB(K-MINSN)=(1-P)*AB(K-MINSN).P.D(K-MINSN)+P-P*AB(K-
          :1MINSN).P.(D(K-MINSN)/C(K-MINSN))
          TRANSFER TO ALPHA
BETA      CONTINUE
          PB(K-MINSN)=1-P
ALPHA     CONTINUE
          PRINT RESULTS Z,P
          EXECUTE OUT.(NAME,PB,MINSN,MAXSN,DELSN)
          VECTOR VALUES NAME=$PB(SN) *$
          EXECUTE CHNCOM.
          END OF PROGRAM
R 1.733+.400
```

```
  PETT     MAD       08/11  1617.3

          DIMENSION C(1000),PA(1000),RR(1000)
          INTEGER K,MINV,MAXV
          EXECUTE IN.($RR(V)*$,RR,MINV,MAXV,DELV)
          SN=VALUE.($SN*$)
          P=VALUE.($P*$)
          A=1/(2*SN+1)
          D=1+1/(2*SN)
          THROUGH ALPHA,FOR K=MINV,1,K.G.MAXV
          AB=P*A/(1-P)
          WHENEVER AB .GE. 1.0, TRANSFER TO BETA
          C(K -MINV)=2*SN*RR(K-MINV).P.2+1
          PA(K-MINV)=(1-P)*(AB.P.D)+P-P*(AB.P.(D/C(K-MINV)))
          TRANSFER TO ALPHA
  BETA    CONTINUE
          PA(K-MINV)=1-P
  ALPHA   CONTINUE
          PRINT RESULTS SN,P
          EXECUTE OUT.($PA(V)*$,PA,MINV,MAXV,DELV)
          EXECUTE CHNCOM.
          END OF PROGRAM
  R  1.350+.266+.000
```

```
ORSCOR    MAD      08/17  2229.0

          DIMENSION R(1000),Y(1000),C(1000),S(1000)
          INTEGER MINV, MAXV, J, I, MINT, MAXT, MINW, MAXW
          EXECUTE IN.($R(V)*$,R,MINV,MAXV, DELV)
          EXECUTE RANGE.($T*$,MINT,MAXT,DELT)
          TO=VALUE.($TO*$)
          TOS=TO.P.2
          THROUGH LOOPA, FOR J=MINT,1,J.G.MAXT
          T=J*DELT
          INTEGA=0
          INTEGB=0
          THROUGH LOOPB, FOR I=MINV,1,I.G.MAXV
          V=DELV*I
          ARG=1.5708*(V+T).P.2/TOS
          INTEGA=INTEGA+R(I-MINV)*COS.(ARG)
LOOPB     INTEGB=INTEGB+R(I-MINV)*SIN.(ARG)
LOOPA     Y(J-MINT)=.70711*SQRT.(DELV.P.2*(INTEGA.P.2+INTEGB.P.2)/TOS)
          EXECUTE OUT.($Y(T)*$,Y,MINT,MAXT,DELT)
          EXECUTE CHNCOM.
          END OF PROGRAM
R  1.100+.383+.000
```

```
PETTY     MAD      08/21  2233.3

          DIMENSION C(1000),HA(1000),YY(1000)
          INTEGER K,MINT,MAXT
          EXECUTE IN.($YY(T)*$,YY,MINT,MAXT,DELT)
          SN=VALUE.($SN*$)
          P=VALUE.($P*$)
          A=1/(2*SN+1)
          D=1+1/(2*SN)
          THROUGH ALPHA, FOR K=MINT,1,K.G.MAXT
          AB=P*A/(1-P)
          WHENEVER AB.GE. 1.0, TRANSFER TO BETA
          C(K-MINT)=2*SN*YY(K-MINT).P.2+1
          HA(K-MINT)=(1-P)*(AB.P.D)+P-P*(AB.P.(D/C(K-MINT)))
          TRANSFER TO ALPHA
BETA      CONTINUE
          HA(K-MINT)=1-P
ALPHA     CONTINUE
          PRINT RESULTS SN,P
          EXECUTE OUT.($HA(T)*$,HA,MINT,MAXT,DELT)
          EXECUTE CHNCOM.
          END OF PROGRAM
R  .733+.210+.000
```

```
r map
'' 1410.7

COMMAND PLEASE
create
TYPE IN COMMANDS, ONE PER LINE.  WHEN ALL COMMANDS HAVE BEEN ENTERED, GIVE TWO
CARRIAGE RETURNS, EDIT IF NECESSARY, AND GIVE COMMAND 'FILE XXXXXX' (WHERE XXXXXX IS
A NAME OF 6 OR FEWER CHARACTERS BY WHICH YOU CAN IDENTIFY YOUR COMMAND SEQUENCE).
THE COMMANDS CAN BE EDITED AND PRINTED BY USING THE CONVENTIONS GIVEN IN THE MANUAL.
TABS NOT SET
INPUT:
(d=1+1/2*sn)?(d=1+1/(2*sn))
(a=1/(2*sn+1))
(z(t)=y(t)/.00022)
(e(t)=2*sn*z(t)*z(t)+1)
(ab=p*a/(1-p))
(pr(t)=(1-p)*ab**d+p*ab**(d/c(t)))

EDIT:
file prob

COMMAND PLEASE
execute correl
PLEASE PRINT ON NEXT LINE MIN, MAX, AND DEL FOR THE VARIABLE      T
.00001 .0001 .000001
MIN =      10       MAX =      99

COMMAND PLEASE
run prob

        COMMAND PLEASE
        EXECUTE PESNR
        DEC. VALUE OF CONSTANT       Z PLEASE.    1.
        DEC. VALUE OF CONSTANT       P PLEASE.    .9

                  Z =      1.000000,              P =       .900000

        COMMAND PLEASE
        (PM(SN)=PB(SN))

        COMMAND PLEASE
        DELETE Z

        COMMAND PLEASE
        EXECUTE PESNR
        DEC. VALUE OF CONSTANT       Z PLEASE.    .671

                  Z =       .671000,              P =       .900000

        COMMAND PLEASE
        (PN(SN)=PB(SN))

        COMMAND PLEASE
        DELETE Z

        COMMAND PLEASE
        EXECUTE PESNR
        DEC. VALUE OF CONSTANT       Z PLEASE.    .324

                  Z  =      .324000,              P =       .900000

        COMMAND PLEASE
        PLOT PM(SN) PN(SN) PB(SN)
```

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RP:dek
8/31/67